St. Name:
Serial\#:

Q1: Suppose that $10 \%$ of the items produced by a certain company is defective. A random sample of 9 items has been selected from the product of this company. Then find:
a) the probability that the sample contains at most 2 defective items.

Solution: Let $X=$ the number of defective items in the sample. Then $X \sim b(9,0.1)$ and $f(x)=(9 C x)(.1)^{x}(0.9)^{(9-x)}, x=0,1, \ldots, 9$.
Then: $\mathrm{P}($ that the sample contains at most 2 defective items $)=\mathrm{P}(\mathrm{X} \leq 2)$ $=\mathrm{f}(0)+\mathrm{f}(1)+\mathrm{f}(2)=(0.9)^{(9)}+9(.1)(0.9)^{(8)}+9 \mathrm{C} 2(.1)^{2}(0.9)^{(7)}$
b) the probability that the sample contains at least 2 defective items given that the sample contains between 1 and 4 defective items .

Solution: P (that the sample contains at least2 defective items given that the sample contains between 1 and 4 defective items $)=P(X \geq 2 / 1 \leq X \leq 4)$
$=\mathrm{P}(\mathrm{X} \geq 2,1 \leq \mathrm{X} \leq 4) / \mathrm{P}(1 \leq \mathrm{X} \leq 4)=\mathrm{P}(2 \leq \mathrm{X} \leq 4) / \mathrm{P}(1 \leq \mathrm{X} \leq 4)$
$=(\mathrm{f}(2)+\mathrm{f}(3)+\mathrm{f}(4)) /(\mathrm{f}(1)+\mathrm{f}(2)+\mathrm{f}(3)+\mathrm{f}(4))$

$$
\begin{aligned}
= & \left(9 \mathrm{C} 2(.1)^{(2)}(0.9)^{(7)}+9 \mathrm{C} 3(.1)^{(3)}(0.9)^{(6)}+9 \mathrm{C} 4(.1)^{(4)}(0.9)^{(5)}\right) / \\
& \left(9(.1)(0.9)^{(8)}+9 \mathrm{C} 2(.1)^{(2)}(0.9)^{(7)}+9 \mathrm{C} 3(.1)^{(3)}(0.9)^{(6)}+9 \mathrm{C} 4(.1)^{(4)}(0.9)^{(5)}\right.
\end{aligned}
$$

c) What is the expected number of defective items in the sample?

Solution: The expected number of defective items in the sample $=\mathrm{E}(\mathrm{X})$ $=(9)(0.1)=0.9$

Q2: Let Z has a standard normal distribution. Then find:
a) $\mathrm{P}(-1.04 \leq \mathrm{Z} \leq 0.75)$

Solution: $\mathrm{P}(-1.05 \leq \mathrm{Z} \leq 75)=\mathrm{A}(1.05)+\mathrm{A}(0.75)$
$=0.3531+0.2734=0.6265$
b) a such that $\mathrm{P}(\mathrm{a} \leq \mathrm{Z})=0.85$

Solution: $\mathrm{P}(\mathrm{a} \leq \mathrm{Z})=0.85$ implies that $: \mathrm{A}(\mathrm{a})+0.5=0.85$. Then $\mathrm{A}(\mathrm{a})=0.35$ Which implies that $\mathrm{a}=-1.04$

