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# King Fahd University Of Petroleum & Minerals
## Mathematical sciences Department

## Second Major exam - Term: 042
## (A)
## Math 131 – Finite Mathematics

**Time allowed: 90 minutes**

| **Name:** | **ID#:** | **Section:** | **Serial:** |
|---|---|---|---|

| Question | Full Mark | Student mark |
|---|---|---|
| 1 | 8 | |
| 2 | 8 | |
| 3 | 8 | |
| 4 | 8 | |
| 5 | 8 | |
| 6 | 10 | |
| Total | 50 | |

**Question 1 (8 points) :**

**Find the dual of the following linear programming problem and then solve it.`**

$Minimize: \quad Z = 8x_1 + 8x_2 + 12x_3$

$subject\ to:$

$2x_1 - 2x_2 - 2x_3 \le 6$

$2x_1 - 2x_2 + 2x_3 \ge 6$

$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$

$\rightarrow \quad -2x_1 + 2x_2 + 2x_3 \ge -6$

$2x_1 - 2x_2 + 2x_3 \ge 6$

$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$

**Then the dual problem is:**

$Maximize: \quad W = -6y_1 + 6y_2$

$subject\ to:$

$-2y_1 + 2y_2 \le 8$

$2y_1 - 2y_2 \le 8$

$2y_1 + 2y_2 \le 12$

$y_1 \ge 0,\ y_2 \ge 0$

**which implies that**

$-2y_1 + 2y_2 + s_1 = 8$

$2y_1 - 2y_2 + s_2 = 8$

$2y_1 + 2y_2 + s_3 = 12$

$y_1 \ge 0,\ y_2 \ge 0, s_1 \ge 0, s_2 \ge 0, s_3 \ge 0$

$and \quad w + 6y_1 - 6y_2 = 0$

**Then:**

$$\begin{bmatrix} -2 & 2 & 1 & 0 & 0 & 0 & 8 \\ 2 & -2 & 0 & 1 & 0 & 0 & 8 \\ 2 & 2 & 0 & 0 & 1 & 0 & 12 \\ 6 & -6 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 1 & 1/2 & 0 & 0 & 0 & 4 \\ 2 & -2 & 0 & 1 & 0 & 0 & 8 \\ 2 & 2 & 0 & 0 & 1 & 0 & 12 \\ 6 & -6 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 1/2 & 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 1 & 0 & 0 & 16 \\ 4 & 0 & -1 & 0 & 1 & 0 & 4 \\ 0 & 0 & 3 & 0 & 0 & 1 & 24 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & \frac{1}{2} & 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 1 & 0 & 0 & 16 \\ 4 & 0 & -1 & 0 & 1 & 0 & 4 \\ 0 & 0 & 3 & 0 & 0 & 1 & 24 \end{bmatrix}$$ **which implies that** $y_1 = 0, s_1 = 0, y_2 = 4, s_2 = 16, s_3 = 4$

**and the maximum value of W equals 24 which is the same as the minimum value of Z**

**Question 2 (8 points):**

**If a principal of $2300 accumulated to $2700 in 4 years at an interest rate which is compounded quarterly, then find:**

**a) the nominal rate of interest. (4 points)**

**Solution:** $S = P\left(1+r\right)^{n}$ **which implies that**

$$2700 = 2300\left(1+r\right)^{16} \rightarrow \frac{2700}{2300} = \left(1+r\right)^{16} \rightarrow In\left(\frac{2700}{2300}\right) = 16In\left(1+r\right)$$

$$\rightarrow In\left(1+r\right) = \frac{In\left(\frac{2700}{2300}\right)}{16} = 0.010 \rightarrow 1+r = e^{0.010} = 1.01005$$

$$\rightarrow r = 0.01005$$

**Which implies that: the nominal interest rate = 4(0.01005)% = 4%.**

**b) the effective rate of this investment. (4 points)**

$$r_e = \left(1+\frac{\text{nominal rate}}{4}\right)^{4} - 1$$

$$= \left(1+\frac{0.04}{4}\right)^{4} - 1 = 4.1\%$$

**Question 3 (8 points):**

a. **If *$19320* is invested for 8 years at an interest rate of 6.5% compounded continuously, then find the compounded amount and compounded interest. (4 points)**

**Solution:**

$$S = Pe^{rt} = 19320e^{(.065)(8)}$$
$$= 19320e^{(.52)} = \$32496.77$$

**The compound interest = S- P**
**= $32496.77 – 19320**
**= $13176.7**

b. **If *$6230* is invested at an interest rate of 6.5% which is compounded continuously, then find the how long does it take the amount to double? (4 points)**

**Solution:**

$$S = Pe^{rt} \quad \rightarrow \quad 2P = Pe^{(.065)t}$$

**which implies that**

$$\rightarrow \quad 2 = e^{(.065)t} \quad \rightarrow \quad In\,2 = (.065)t$$
$$\rightarrow t = \frac{In\,2}{.065} = 10.66 \ \ years$$

**Question 4 (8 points):**

a. **A debt consists of *$3550* due in three years from now and *$6250* due in seven years from now is to be repaid by a payment of *$2000* in one years and *$3000* in three years and a final payment at the end of six years. If the interest rate is 9% compounded semiannually, how much should be the final payment? (4 points)**

**Solution:**

$$3550(1.045)^{-6}+6250(1.045)^{-14}=2000((1.045)^{-2}+3000(1.045)^{-6}+x\,(1.045)^{-12}$$
$$\rightarrow\ 2726.30+3374.83=1831.46+2303.69+0.59x$$
$$\rightarrow\ x=\frac{2726.30+3374.83-1831.46-2303.69}{0.59}$$
$$\rightarrow\ x=\$3332.17$$

b. **An initial investment of $30000 in a project guarantees the cash flows of $10000 after 3 years, $12000 after 5 years and $14000 after 6 years. If the interest rate is 6% compounded semiannually, then determine whether the investment is profitable or not? (4 points)**

**Solution:**

$$\begin{aligned}The\ NPV &=10000(1.03)^{-6}+12000(1.03)^{-10}+14000((1.03)^{-12}-30000\\ &=8374.84+8929.13+9819.32-30000\\ &=-\$2876.71\end{aligned}$$

**Therefore, the investment is not profitable.**

**Question 5 (8 points):**

a) **An annuity of equal payments at the end of each quarter for 3 years is to be purchased of $16000 . If the interest rate is 8% compounded quarterly then determine how much is each of the payments. (4 points)**

**Solution:**

$$A = R\, a_{n,r} \quad \text{, where } a_{n,r} = \frac{1-(1+r)^{-n}}{r} = \frac{1-(1+.02)^{-12}}{.02} = 10.58$$

**implies that**

$$R = \frac{A}{a_{n,r}} = \frac{16000}{10.58} = \$1512.29$$

b) **An annuity of equal payments at the beginning of each month is worth $12000 after for 5 years. If the interest rate is 6% compounded monthly then determine how much is each of the payments. (4 points)**

**Solution:**

$$S = R\,[s_{n+1,r} - 1] \quad \text{, where } s_{n+1,r} = \frac{(1+r)^{(n+1)} - 1}{r} = \frac{\left(1+\frac{.06}{12}\right)^{(60+1)} - 1}{\frac{.06}{12}} = 71.12$$

**implies that**

$$R = \frac{S}{s_{n+1,r} - 1} = \frac{12000}{71.12 - 1} = \$171.14$$

**Question 6 :( 10 Points)**

a) **In how many ways we can order a group of 6 men and 4 women in a line if the women are to stand in the back of the line? (3 points)**

**Solution:**

**(6!)(4!)**

b) **How many words can be formed the letters of the word 'statistics'? (3 points)**

**Solution:**

$$\frac{10!}{3!3!2!}$$

c) **Draw 5 cars at random from a deck of 52 playing cards without replacement. Then find the probability of having at least one even number. (4 points)**

**Solution:**

P(at least one even number) = 1 - P(no even numbers)

$$= 1 - \frac{26C5}{52C5}$$