King Fahd University Of Petroleum & Minerals
Mathematical sciences Department

Term: 042

Math 131 – Finite Mathematics

Time allowed: 90 minutes

Name: ID#: Section: Serial:

Question	Full Mark	Student mark
1	5	
2	5	
3	10	
4	10	
5	10	
6	10	
Total	50	

Question 1 :(5 Points)

An investor wants to invest \$50,000 in two companies for one year. The annual returns of the two companies are 6% and 10% respectively. **Find** the amounts of money that he will invest in the two companies if the total annual return is equivalent to an annual return of 7% on the entire amount.

Solution:

Let x = The amount that will be invested in the 1st company. Thus \$(50,000-x) will be invested in the 2nd company. Then:

$$0.06x + 0.1(50,000 - x) = 0.07(50,000) \Rightarrow 0.06x - 0.1x + 5,000 = 3,500$$

$$\Rightarrow -0.04x = 3,500 - 5,000 = -1,500$$

$$x = \frac{-1,500}{-.04} = 37,500$$

Thus \$37,500 will be invested in the 1st company and \$12,500 will be invested in the 2nd company.

Question 2 :(5 Points)

A publisher sells a magazine for \$3 per one and costs him \$2.2 per one. In addition, he earns 10% of the total sale from advertisement. **What** is the minimum number of magazines that he should sell to get at least \$1200 as a profit?

Solution:

Let x = The number of magazines that he will sell. Then:

$$\text{Profit} = TR - TC = 3x + 0.1(3x) - 2.2x = 3.3x - 2.2x = 1.1x$$

Then

$$Profit \geq 1,200 \text{ iff } 1.1x \geq 1,200 \text{ iff } x \geq \frac{1200}{1.1} = 1090.91,$$

Therefore, he should sell at least 1,091 to get at least \$1200 as a profit.

Question 3 :(10 Points)

Find the equation of the line that passes through (2, 1) and perpendicular to the line $3y + 6x = 12$. **(5 Points)**

Solution:

The slope of the line $3y + 6x = 12$ is (-2). Therefore, the slope of the required line is 1/2. Then the equation of the required line is given by:

$$y - 1 = \left(\frac{1}{2}\right)(x - 2)$$

which implies that

$$y = \frac{x}{2}$$

a) The demand function for a certain product is $p = 2400 - 3q$, where p the price in dollars per unit is and q is the number of units demanded per month. **Find** the number of units that maximizes the total revenue and **determine** the maximum total revenue. **(5 Points)**

Solution:

$$TR = pq = (2,400 - 3q)q = 2,400q - 3q^2$$

The TR is a quadratic function in q. The vale of q which maximizes TR is given by:

$q = \frac{-2,400}{2(-3)} = 400$, which implies that the maximum total revenue is:

$$2,400(400) - 3(400)^2 = \$480,000.$$

Question 4 :(10 Points)

A producer sells his product at \$32 per unit. If the fixed costs of his product are \$8,200 and the variable costs are \$20 per unit. Assume that he sells his entire product, then:

a. **Determine** the breaking even point **(6 Points)**

Solution:

Let x = The number of units that he will sell. Then:

$$\text{Profit} = P = TR - TC = 32x - (8,200 + 20x) = 12x - 8,200$$

Then: $\text{Profit} = 0$ iff $12x = \frac{8,200}{12}$ iff $x = 683.33$

Therefore, the breaking even point quantity is 683.33 units and the breaking even total revenue is \$12(683.33)=\$8,199.96, which implies that the breaking even point is (683.33,8199.96).

b. If the total costs have increased 10%, then **find** the level of production at the new breaking even point. **(4 Points)**

Solution:

$$\text{Profit} = P = TR - TC = 32x - (8,200 + 20x)(1.1) = 10x - 9,020$$

Then: $\text{Profit} = 0$ iff $10x = 9,020$ iff $x = 902$

Thus 902 is the breaking even quantity.

Question 5 :(10 Points)

a) **Solve** the following system of linear equations **by matrix reduction**: **(6 points)**

$$\begin{aligned} 2x - 2y - 6z &= -10 \\ 6x - 3y - 12z &= -24 \\ -x - y + z &= 1 \end{aligned}$$

Solution:

$$\begin{bmatrix} 2 & -2 & -6 & -10 \\ 6 & -3 & -12 & -24 \\ -1 & -1 & 1 & 1 \end{bmatrix} (R_1/2 \; and \; R_2/3)$$

$$\begin{bmatrix} 1 & -1 & -3 & -5 \\ 2 & -1 & -4 & -8 \\ -1 & -1 & 1 & 1 \end{bmatrix} (-2R_1 + R_2 \; and \; R_1 + R_3)$$

$$\begin{bmatrix} 1 & -1 & -3 & -5 \\ 0 & 1 & 2 & 2 \\ 0 & -2 & -2 & -4 \end{bmatrix} (R_1 + R_2 \; and \; 2R_2 + R_3)$$

$$\begin{bmatrix} 1 & 0 & -1 & -3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix} (R_3/2)$$

$$\begin{bmatrix} 1 & 0 & -1 & -3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} (R_3 + R_1 \; and \; -2R_3 + R_2)$$

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Thus $x = -3$, $y = 2$ and $z = 0$

b) **Solve** the following nonlinear system of equations: **(4 points)**

$$x^2 = 5 + y \qquad (1)$$
$$x = y - 1 \qquad (2)$$

Solution:

From (2), y = x+1. Then substitute this value of y in (1) we get:

$x^2 = 5 + x + 1 = x + 6$

Then it follows that: $x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0$

Which implies that x =3 or x=-2. If x=3, then y=4 and if x=-2, then y=-1. Therefore, the solutions of the system are: (3,4) and (-2,-1).

Question 6:(10 Points)

Find x and y, which maximize Z geometrically:

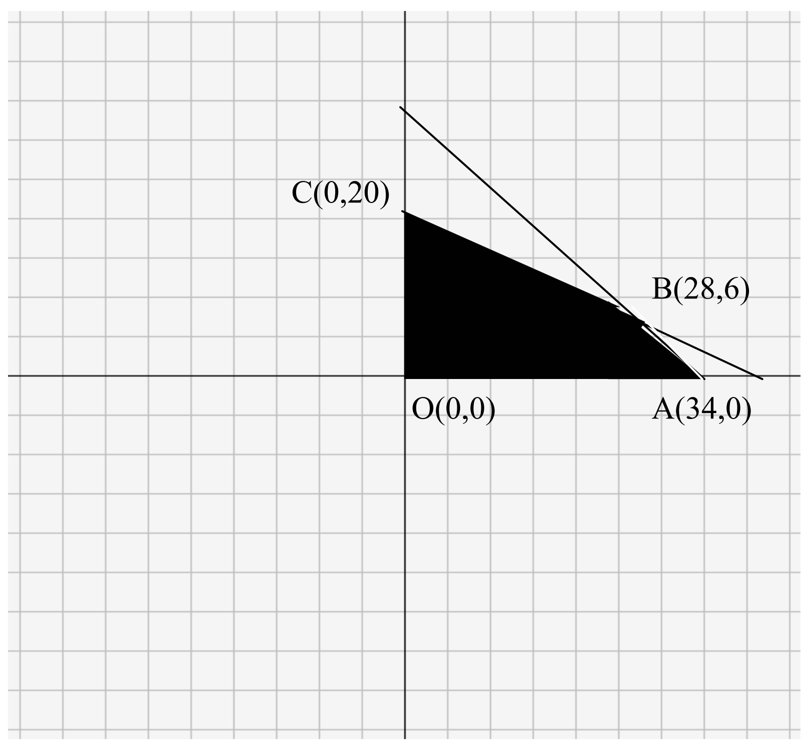
$Z = 10x + 15y$

subject to :

$$x + 2y \le 40$$
$$-x - y \ge -34$$
$$x, y \ge 0$$

and **find** the maximum value of Z.

Solution:



The feasible region is the black region in the figure with corner points O, A, B and C. The value of Z at these points are: 0, 340, 370 and 300 respectively. Thus, B maximizes Z and the maximum value of Z will be 370.