1. Let $f(x) = \ln(1+x)$, $x_0 = 1$ and $x_1 = 1.1$. Use linear interpolation to calculate an approximation value for $f(1.04)$, and obtain a bound on the truncation error.

2. Use the Newton-divided difference polynomial to approximate $f(3)$ from the following table:

| $x$ | 0 | 1 | 2 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 14 | 15 | 5 | 6 | 19 |

3. The following is the tabulation of an actual thermodynamic quantity:

| $x$ | 0 | 0.2 | 0.4 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 1.000 | 0.916 | 0.836 | 0.0741 | 0.624 | 0.429 | 0.224 | 0.265 | 0.291 | 0.316 |

Approximate $f(0.23)$ using the Lagrange interpolating polynomial.

4. Given the following tabulated function

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f(x)$ | $-7$ | $-4$ | 5 | 26 | 65 | 128 |

This tabulated function is a polynomial. Find the degree of the polynomial and the coefficient of the highest power of $x$.

5. Use Lagrange interpolation, find $f(4.4)$ for the following function:

| $x$ | 0 | 1.0 | 2.0 | 3.8 | 5.0 |
|---|---|---|---|---|---|
| $f(x)$ | 0 | 0.569 | 0.792 | 0.223 | $-0.186$ |

6. The following table gives the sine for angles at an interval of $2^o$ :

| $\sin\theta$ | 0.000 | 0.035 | 0.068 | 0.105 | 0.140 | 0.174 | 0.208 | 0.242 | 0.276 |
|---|---|---|---|---|---|---|---|---|---|
| Angle $\theta^o$ | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |

   (a) Using the Newton-divided difference polynomial to estimate sine of $9^o$. Compare with the exact value $\sin(9^o) = 0.1564344651$.

   (b) Derive an interpolating polynomial using an increment of $4^o$ and estimate the sine of $9^o$. Compare the error with the error for part (a).

7. Determine an appropriate step size to use, in the construction of a table of $f(x) = (1+x)^6$ on $[0, 1$. The truncation error for linear interpolation is to be bounded by $5 \times 10^{-5}$.

8. Determine the maximum step size that can be used in the interpolation of $f(x) = e^x$ in $[0, 1]$,so that the error in the linear interpolation will be less than $5 \times 10^{-4}$. Find also the step size if quadratic interpolation is used.

9. Find the unique polynomial $p(x)$ of degree 2 or less such that $p(1) = 1$, $p(3) = 27$, $p(4) = 64$ using each of the following methods.

   (a) The Lagrange interpolating polynomial.

(b) The Newton-divided difference polynomial.

10. Suppose that $f(x) = e^x \cos x$ is to be approximated on $[0, 1]$ by an interpolating polynomial on $n + 1$ equally spaced points $0 = x_0 < x_1 < ... < x_n = 1$. Determine $n$ so that the truncation error will be less than 0.0001 in this interval.

11. Given the function $f(x) = 1/(1 + x^2)$

    (a) Find the Lagrange polynomial $p(x)$ that interpolates $f$ at the points $x = 0, 1, 3, 5$.
    (b) Compute $p(4)$ and compare the result with $f(4)$.
    (c) Find the fourth derivative of $f$.
    (d) Evaluate the error $f(4) - p(4)$.
    (e) Find the polynomial $q$ such that $f(0) = q(0)$, $f'(0) = q'(0)$, $f(1) = q(1)$ and $f'(1) = q'(1)$.
    (f) Compare $f(4)$ and $q(4)$.

12. Show that if $g$ interpolates $f$ at the points $x_1, x_2, ..., x_{n-1}$ and $h$ at the points $x_2, ..., x_n$, then

$$k(x) = g(x) + \frac{x_1 - x}{x_n - x_1}[g(x) - h(x)]$$

interpolates $f$ at the points $x_1, x_2, ..., x_n$.

13. (Continuation) Show that if $f$ is continuously differentiable in $[x_1, x_2]$ then

$$\text{there exists } c \in (x_1, x_2) \text{ such that } f[x_1, x_2] = f'(c)$$

## Computer assignment

1. Use the MATLAB function newtondd.m to find the polynomial that interpolates the function $e^x$ at 11 equally spaced points in the interval $[0, 1]$. Compare the values of polynomial to $e^x$ at the midpoints.

2. Use the MATLAB function lagrange.m to find the value of interpolating polynomial at $x = 1.6$ that assumes these values:

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | 2.5 | 10 | 5 | 1 | 1 | 0 |

3. Use the MATLAB function newtondd.m to find the polynomial that interpolates the function $\tan x$ at 21 equally spaced points in the interval $[0, \pi/4]$.

4. Use the MATLAB function lagrange.m to find the value of interpolating polynomial at $x = 0.9$ in Exercise 3.