1. Use the bisection method to find the root of the equation $x + \cos x = 0$ correct to two decimal places.

2. Use the bisection method to find to three decimal places the positive root of the equation $x - 0.2 \sin x - 0.5 = 0$

3. The function $f(x) = x^4 - 8.6x^3 - 35.51x^2 + 464.4x - 998.46$ has a simple root in the interval $[6, 8]$ and a double root in the interval $[4, 5]$. Use the bisection method to find both roots.

4. Find to four decimal places a root of $0.1x^2 - x \ln x$ between 1 and 2.

5. The function $f(x) = x^2 - 0.8x - 8.3$ has a root in $[2, 4]$. How many bisections would be required to locate this root to an accuracy of 10^{-5}?

6. Show that formula (3.6) for the false position method is algebraically equivalent to

$$c_n = \frac{f(b_n)a_n - f(a_n)b_n}{f(b_n) - f(a_n)}$$

7. Find the interval in which the smallest positive root of the following equations lies:

   (a) $\tan x + \tanh x = 0$

   (b) $x^3 - x - 4 = 0$

   Determine the roots correct to three decimals using the false position method

8. Find the real roots of $f(x) = x^3 - 2.56x^2 - 34.6x + 112.5$ using the method of false position.

9. Find the smallest positive root of $f(x) = \cos x \cosh x - 1$.

10. The function $f(x) = x^2 - 2e^{-x}x + e^{-2x}$ has one multiple real root. Use the modified Newton's method to approximate to within 10^{-4} this root with $x_0 = 1$. Also try to find the root using Newton's method with the same initial guess and compare the number of iterations needed to attain the same accuracy.

11. The function $f(x) = x^4 - 8.6x^3 - 35.51x^2 + 464.4x - 998.46$ a double root in the interval $[4, 5]$. Use the modified Newton's method to find the double root.

12. Determine the order of convergence of the iterative method

$$x_{k+1} = (x_0 f(x_k) - x_k f(x_0)/(f(x_k) - f(x_0)).$$

for finding the root of the equation $f(x) = 0$.

13. Find the iterative methods based on Newton's method for finding $\sqrt{n}$, $\sqrt[3]{n}$, and $1/n$ where n is a real number. Apply the methods to $n = 18$ to obtain the results correct to two decimals.

14. Given the following equations

    (a) $x^4 - x - 10 = 0$

    (b) $x - e^{-x} = 0$

    Determine the initial approximations. Use these to find the roots correct to four decimal places using the secant method.

15. Show that formula (3.7) for the secant method is algebraically equivalent to

$$x_{n+1} = \frac{f(x_n)x_{n-1} - f(x_{n-1})x_n}{f(x_n) - f(x_{n-1})}$$

16. Use the secant method to compute the next two iterates x_2 and x_3 for the zeros of the following functions.

    (a) $f(x) = x^2 - 3x - 1$ with $x_0 = 3.0, x_1 = 3.1$

    (b) $f(x) = x^3 - x - 1$ with $x_0 = 1.2, x_1 = 1.3$

    (c) $f(x) = x^3 - 2x + 1$ with $x_0 = 0.6, x_1 = 0.5$

17. Show that the following two sequences have convergence of the order with the same limit $\sqrt{a}$.

    $x_{n+1} = \frac{1}{2}x_n(1 + a/x_n^2)$

    $x_{n+1} = \frac{1}{2}x_n(3 - x_n^2/a)$

    If x_n is a suitably close approximation to $\sqrt{a}$, show that the error in the first formula for x_{n+1} is about one third of that in the second formula, and deduce that the formula

    $x_{n+1} = \frac{1}{8}x_n(6 + 3a/x_n^2 - x_n^2/a)$

    gives a sequence with third-order convergence.

18. Given the equation $f(x) = 0$, obtain an iterative method using the rational approximation

$$f(x) = \frac{x - a_0}{b_0 + b_1 x}$$

    where the coefficient a_0, b_0, and b_1 are determined by evaluating $f(x)$ at x_k, x_{k-1} and x_{k-2}. Carry out two iterations using this method for the equation $2x^3 - 3x^2 + 2x - 3 = 0$ with $x_0 = 0$, $x_1 = 1$ and $x_2 = 2$.

    (a) Show that the equation $\ln x = x^2 - 1$ has exactly two real roots, $\alpha_1 = 0.45$ and $\alpha_2 = 1$.

(b) Determine for which initial approximation x_0, the iteration
$x_{n+1} = \sqrt{1 + \ln x_n}$
converges to α_1 or α_2.

(c) Determine for which initial approximation x_0, the iteration
$x_{n+1} = x_n - \frac{x_n - \exp(x_n^2 - 1)}{1 - 2x_n \exp(x_n^2 - 1)}$
converges to a root of the equation in (a) and in case of convergence also which root.

19. show that x-coordinate of the point on the graph of $y = x^2 + 1$ that is closest to the point $(3, 1)$.is given by the solution of the equation $d'(x) = f(x) = 0$ where $d(x) = (x-1)^2 + (x^2-2)^2$. Find the root of $f(x) = 0$ using Newton's method with $x_0 = 1$

20. Apply Newton's method with $x_0 = 0.8$, and the secant method with $x_0 = 0.8$, $x_1 = 1.2$ to the equation
$x^3 - x^2 - x + 1 = 0$
and verify that the convergence is only of the first order in each case.

21. Solve he nonlinear system of equations

$$\begin{aligned} \ln(x^2 + y) - 1 + y &= 0 \\ \sqrt{x} + xy &= 0 \end{aligned}$$

using Newton's method with $x_0 = 2.4$ and $y_0 = -0.6$.

22. Solve he nonlinear system of equations

$$\begin{aligned} 10 - x + \sin(x + y) - 1 &= 0 \\ 8y - \cos^2(z - y) - 1 &= 0 \\ 12z + \sin z - 1 &= 0 \end{aligned}$$

using Newton's method with $x_0 = 0.1$, $y_0 = 0.25$, and $z_0 = 0.08$

Computer assignment

1. Use the MATLAB function bisect.m to approximate a root of the equation

$$x^4 - 8.6x^3 - 35.51x^2 + 464.4x - 998.46 = 0$$

to four correct decimals in the interval $[6, 8]$.

2. Use the MATLAB function false.m to find the points of intersection of the functions
$f(x) = x^4 - 2x^3 - 5x^2 - 6x - 24$ and $g(x) = x^5 + 2x^3 - 4x^4 - 8x^2$ located in the interval $[3, 5]$.

3. The largest positive root of the equation

$$(x+3)(x^2-1)^8 = 3 \times 10^{-8} x^{15}$$

is to be computed. On a PC which uses floating arithmetic and eight decimals, the equation, the standard form,

$$x^{17} + 3x^{16} - (8 + 3 \times 10^{-8})x^{15} + ... + 3 = 0,$$

is stored in the PC as

$$x^{17} + 3x^{16} - 8.0000000\, x^{15} + ... + 3 = 0.$$

Thus the PC will treat the equation $(x+3)(x^2-1)^8 = 0$, whose exact positive root is 1 which is a poor result. One can get the root to full PC accuracy by writing the equation in the form

$$x = 1 + \frac{1}{x+1}\frac{1}{10}\left(\frac{3x^{15}}{x+3}\right)^{1/8}$$

and solving this by using Newton's method with $x_0 = 1$

4. Use the MATLAB function secant.m to approximate a smallest positive root of the equation

$$f(x) = 34x^8 - 456x^7 + 33x^5 - 212x^4 + 789x^3 - 45x + 345 = 0$$

with $x_0 = 1$, and $x_1 = 2$

5. Write a computer program in a language of your choice to find the positive root of the equation

$$\cosh x + \cos x - c = 0$$

for $c = 4, 3, 2, 1$, using the secant method with an accuracy of 10^{-5}.

6. Use the MATLAB function newton.m to approximate a positive root of the equation

$$\ln(x^2+1) - e^{-2x^2} - 3\cos(x+1) + \sin x^2 = 0$$

with $x_0 = 1$.

7. Use both the MATLAB functions newton.m and newton2.m to find the root α of order n for the following:

   (a) $f(x) = (x-3.1)^4$, $n = 4$, $\alpha = 3.1$; with $x_0 = 2$.

   (b) $f(x) = (x^3-8)^5$, $n = 5$, $\alpha = 2$; with $x_0 = 1$.

   (c) $f(x) = (x-1)\,e^x$, $n = 1$, $\alpha = 1$; with $x_0 = 0$.

(d) $f(x) = (x-4)^3(x-1)$, $n = 3$, $\alpha = 4$; with $x_0 = 3$.

(e) $f(x) = (x - \frac{\pi}{2})(\sin x - \frac{2}{\pi}x)$, $n = 2$, $\alpha = \frac{\pi}{2}$; with $x_0 = 0.6$.

Compare the number of iterates nedded for each M.function to converge with an accuracy of 10^{-4}.

8. The equation

$$f(x) = x^6 + 8.3x^4 - 6.8x^5 + 24.92x^3 - 51.48x^2 + 35.64x - 65.34$$

has one root of multiplicity 3. Use the MATLAB function newton2.m to approximate this root. Use $x_0 = 2$.

9. One wants to solve the equation $x + \ln x = 0$, whose roots is $\alpha \approx 0.5$, by using the fixed point iteration method. Use the formulas

(a) $x = -\ln x$

(b) $x = e^{-x}$

(c) $x = \frac{x+e^{-x}}{2}$

and see which of the formulas give convergence.

10. Write a computer program in a language of your choice that finds the zeros of the equation

$$e^{-0.2x}\sin(x^2 - 1) + 0.45x = \sqrt{k/2}$$

using Newton's method for the following values of k:

(a) $k = 1$ in $[0, 2]$

(b) $k = 3$ in $[0, 2]$

(c) $k = 5$ in $[2, 3]$

11. Write a computer program in a language of your choice that finds the only real root of the equation

$$103.50x^7 - 530.96x^6 + 25.30x^5 - 129.79x^4 + 103.50x^3 - 530.96x^2 + 25.30x - 129.79 = 0$$

using both the bisection method to produce a sufficiently small interval that contains the root and Newton's method to find the approximate root quickly and with a better accuracy.