1. Solve the following boundary-value problems using a finite difference method
2. $y'' + 9y = 0, \quad y(0) = 4,\ y(2) = 1;\ h = 1/8$
3. $y'' - y = x^2, \quad y(0) = 0,\ y(1) = 0;\ h = 1/4$
4. $y'' + 2y' + y = 5x, \quad y(0) = 0,\ y(1) = 0;\ h = 1/5.$
5. $y'' - 10y' + 25y = 1, \quad y(0) = 1,\ y(1) = 0;\ h = 1/5$
6. $y'' - 4y' + 4y = (x+1)e^{2x}, \quad y(0) = 3,\ y(1) = 0;\ h = 1/5$
7. $y'' + 5y' = 4\sqrt{x}, \quad y(1) = 1,\ y(2) = -1;\ h = 1/5$
8. $x^2y'' + 3xy' + 3y = 0, \quad y(1) = 5, y(2) = 0;\ h = 1/8$
9. $x^2y'' - xy' + y = \ln x, \quad y(1) = 0,\ y(2) = -2;\ h = 1/10.$
10. $y'' + (1-x)y' + xy = x, \quad y(0) = 0,\ y(1) = 2;\ h = 1/10$
11. $y'' + xy' + y = x, \quad y(0) = 1,\ y(1) = 0;\ h = 1/10$
12. $y'' + 6.55(1+x)y = 1, \quad y(0) = 0,\ y(1) = 0;\ h = 1/10.$
13. The electrostatic potential u between two concentric spheres of radius $r = 1$ and $r = 4$ is determined from

$$\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} = 0, \quad u(1) = 50, \quad u(4) = 100.$$

Use the finite difference method decribed in section 10.1 with $n = 12$ to approximate the solution of the BVP.

14. Consider the BVP $y'' + xy = 0,\ y(0),\ y(1) = -1$. Find the difference equation corresponding to the differential equation.

Solve the following boundary-value problems using the shooting method

15. $y'' = y' - \sin(xy), \quad y(0) = 1,\ y(1) = 1.5;\ h = 1/10$
16. $y'' = \frac{1}{2}y - \frac{2(y')^2}{y}, \quad y(0) = 1,\ y(1) = 1.5;\ h = 1/10$
17. $y'' = yy' + e^x, \quad y(0) = 1,\ y(1) = -1;\ h = 1/5$
18. $y'' + xy' + y = x, \quad y(0) = 1,\ y(1) = 0;\ h = 1/10$
19. $y'' = 2 - \frac{4y^2}{\sin^2 x}, \quad y(1) = 0\ .708\,07,\ y(2) = 0.\,826\,82;\ h = 1/10$
20. $y'' = 2y^3, \quad y(2) = \frac{1}{5},\ y(4) = \frac{1}{7};\ h = 1/10$
21. $y'' = -(y')^2 - y + \ln x, \quad y(2) = \ln 2,\ y(4) = 2\ln 2;\ h = 1/10$

Computer Assignements

1. Use the MATLAB function finitediff.m to approximate the solution of Exercise 6.

2. Use the MATLAB function lshoot.m to approximate the solution of Exercise 8.

3. Use the MATLAB function lshoot.m to approximate the solution of the BVP

$$x^2y'' = -4xy' + 2y - 2\ln x, \quad y(1) = -\frac{1}{2},\ y(2) = \ln 2, \quad 1 \leq x \leq 2$$

   with $h = 0.01$