Ex. 13.6

Problem 1. (Revenue) The demand function for a manufacturer's product is given by $q = 10000e^{-0.02p}$, find the level of production that maximizes revenue.

Solution. Demand function: $q = 10000e^{-0.02p}$ implies that

$$p = 50(\ln 10000 - \ln q).$$

Thus total revenue (T_R) is

$$T_R = 50q(\ln 10000 - \ln q) = 50q\ln 10000 - 50q\ln q$$

$$\Rightarrow \quad \frac{dT_R}{dq} = 50\ln 10000 - 50\ln q - 50q(1/q) = 0$$

$$\Rightarrow q = 10000/e \cong 3679.$$

Moreover, since

$$\frac{d^2T_R}{dq^2} = -50/q < 0 \text{ (when } q = 3679\text{)}$$

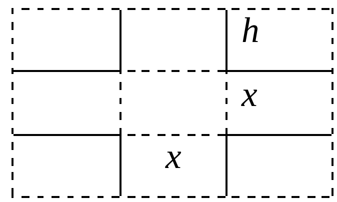
it follows (from SDT) that there is a relative max. when $q = 3679$, however, as it is the only relative extremum on $(0, +\infty)$ it must also be an absolute extremum. *Ans*. $q = 3679$.

<u>Problem 2</u>. (Container Design) An open-top box is a square base is to be constructed with 192 ft^2 of material. What should be the dimensions of the box if the volume is to be a maximum? What is the maximum volume?

<u>Solution</u>.

Surface Area (S): $S = x^2 + 4xh = 192$

$$\Rightarrow \quad h = \frac{192 - x^2}{4x}$$



$(x>0)$

Volume (V): $V = x^2 h = x^2 \frac{(192 - x^2)}{4x}$

$$= \frac{1}{4}(192x - x^3)$$

$$\Rightarrow \quad \frac{dV}{dx} = \frac{1}{4}(192 - 3x^2) = \frac{3}{4}(64 - x^2) = 0$$

$$\Rightarrow \text{x} = 8, -8 \text{ (rejected)}$$

Moreover, since

$$\frac{d^2V}{dx^2} = -9x/2 < 0 \text{ (when } x = 8\text{)}$$

it follows (from SDT) that there is a relative max. when $x = 8$, however, as it is the only relative extremum on $(0, +\infty)$ it must also be an absolute extremum. *Ans.* $x = 8$ *ft*, $h = \frac{192 - 8^2}{4(8)} = 4$ *ft and* $V = 8^2(4) = 256$ ft^3.