King Fahd University of Petroleum and Minerals
Dhahran 31261
Department of Mathematical Sciences

MATH 131 – 04 & 05 (041)
Major Exam II
December 14th , 2004
Instructor: Dr. A. Umar

Duration	1hr 40 mins
Time	17.15-18.55 hrs

SOLUTION.

Name:_____

ID Number:_____ Section Number:_____

Notes:

1. Students must have a valid KFUPM ID Card with them.
2. You must show all your work to justify your answers.
   Be as organized as possible. Only neat and logical solutions will attract points.
3. Programmable calculators and mobile phones are NOT allowed.
4. A scratch paper is attached at the end of this question paper. **PLEASE, DO NOT REMOVE IT.**

Page	Points
2	
3	
4	
5	
6	
Total	

ANSWER ANY FOUR QUESTIONS. Each question carries 25 points.

Q1. Maximize: $Z = x_1 + x_2 + 2x_3$,
subject to

$$x_1 + 2x_2 + x_3 \le 7$$
$$2x_1 + 3x_2 - x_3 \le 1$$
$$x_1 + 3x_2 + x_3 \le 8$$
$$x_1, x_2, x_3 \ge 0.$$

(Indicate: evc, dvr, pivot, quotients, eros, indicators, Z_{max} and the corresponding values of the x_i's)

Add Slack Variables.

$$x_1 + 2x_2 + x_3 + s_1 = 7$$
$$2x_1 + 3x_2 - x_3 + s_2 = 1$$
$$x_1 + 3x_2 + x_3 + s_3 = 8$$
$$-x_1 - x_2 - 2x_3 + Z = 0$$

TABLEAU I

pivot (1)

dvr (1) →

	x_1	x_2	x_3	s_1	s_2	s_3	Z		Quotients
s_1	1	2	+1	1	0	0	0	7	$7/1 = 7$
s_2	2	3	-1	0	1	0	0	1	*
s_3	1	3	1	0	0	1	0	8	$8/1 = 8$
Z	-1	-1	-2	0	0	0	1	0	

(3)

↑ evc (1)

$R_2 \to +R_1 + R_2$
$R_3 \to -R_1 + R_3$
$R_4 \to 2R_1 + R_4$
(3)

x_3	1	2	1	1	0	0	0	7
s_2	3	5	0	1	1	0	0	8
s_3	0	1	0	-1	0	1	0	1
Z	1	3	0	2	0	0	1	14

All indicators are nonnegative. (+2)

The solution is when (3)

$$s_1 = x_1 = x_2 = 0 \; ; \; x_3 = 7, \; s_2 = 8, \; s_3 = 1$$

(3)

& $Z_{max} = 14$. (2)

Q2.

(a) Over a five-year period, an original principal of SR 3000 accumulated to SR 4200 in an account in which interest was compounded bimonthly (every two months). Determine the effective rate of interest, rounded to two decimal places.

[You may use: $\sqrt[30]{1.4} \cong 1.0112789$; or $\sqrt[10]{1.4} \cong 1.03422$; or $\sqrt[5]{1.4} \cong 1.06961$.] [13 points]

$P = 3000,\ S = 4200,\ n = 6 \times 5 = 30,\ r_p = ?$

$4200 = 3000(1 + r_p)^{30}$

$1.4 = (1 + r_p)^{30}$ (8)

$\sqrt[30]{1.4} - 1 = r_p$

$0.0112789 \approx r_p$ (periodic rate).

Effective Rate

$r_e = (1 + r_p)^{n_e} - 1 = (1.0112789)^6 - 1$ (5)

≈ 0.069611 or 6.96%.

$= 0.07$ or 7% (2 d.p.)

(b) If interest is compounded continuously, at what annual rate will a principal of SR 987 quadruple (i. e. four-fold or four times) in 15 years. Give your answer to the nearest percent. (Leave your answer as $\frac{\ln a}{b}$.) [12 points]

$P = 987,\ S = 4(987),\ t = 15.$

$4(987) = 987\, e^{15r}$

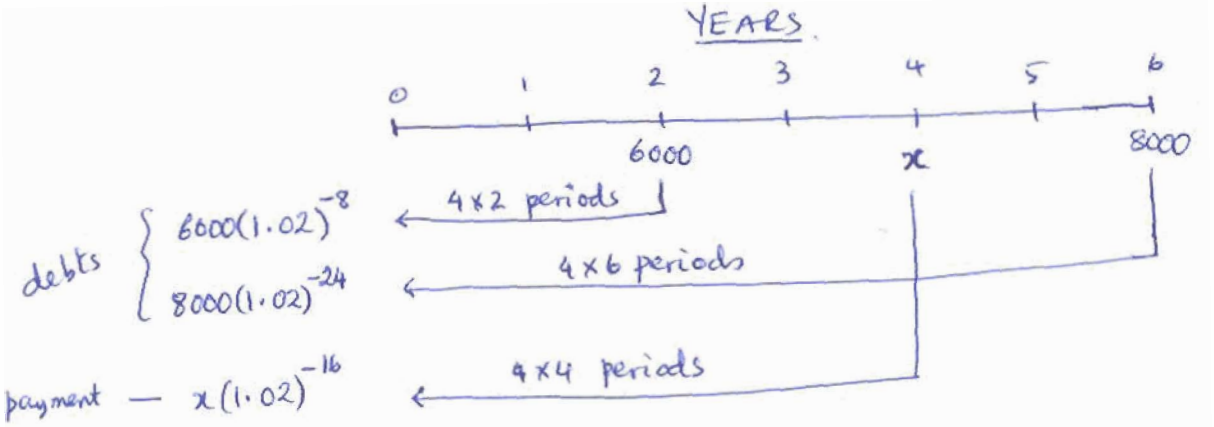
$4 = e^{15r}$

$\ln 4 = \ln e^{15r} = 15r \ln e^{1}$

$\frac{\ln 4}{15} = r$

Q3.

(a) A debt of SR 6000 due in two years and SR 8000 due in six years are to be repaid by a single payment four years from now. If the interest rate is 8% compounded quarterly, how much is the payment? **[12 points]**



$r_p = \frac{0.08}{4} = 0.02$

Equation of Value:

$$x(1.02)^{-16} = 6000(1.02)^{-8} + 8000(1.02)^{-24} \quad (8)$$

$$x = 6000(1.02)^{8} + 8000(1.02)^{-8}$$

$$= 1000(1.02)^{-8}\left[6(1.02)^{16} + 8\right]$$

$$\approx 13857.88 \text{ SR}. \quad (4)$$

(b) An annuity consisting of equal payments at the end of each month for three years is to be purchased now for SR 40 000. If the interest rate is 9%, how much is each payment? **[13 points]**

ORDINARY ANNUITY

$A = 40{,}000$; $n = 3 \times 12 = 36$, $r_p = \frac{0.09}{12} = 0.0075$.

$$40000 = R\,a_{\overline{36}|0.0075}$$

$$R \approx \frac{40000}{31.446805} \approx 1271.99 \text{ SR}.$$

Q4.

(a) Find the present value of SR 800 paid at the beginning of each six-month period for 6 years at the rate of 4% compounded semiannually. [10 points]

ANNUITY DUE

$R = 800$, $n = 2 \times 6 = 12$, $r_p = \frac{0.04}{2} = 0.02$

$$A = 800[a_{\overline{n-1}|r_p} + 1]$$
$$= 800[a_{\overline{11}|0.02} + 1]$$
$$\approx 800[10.786848]$$
$$= 8629.48 \text{ SR}$$

(b) Suppose SR 2000 is placed in a savings account at the beginning of each month for two years. If no further deposits are made, (i) how much is in the account after two years? (ii) How much of this amount is compound interest? Assume that the savings account pays 6% compounded monthly. [15 points]

Annuity Due

$R = 2000$, $n = 12 \times 2 = 24$, $r_p = \frac{0.06}{12} = 0.005$

(i) $$S = R[S_{\overline{n+1}|r_p} - 1] = 2000[S_{\overline{25}|0.005} - 1]$$
$$\approx 2000[25.559115]$$
$$= 51{,}118.23 \text{ SR.}$$ (12)

(ii) Compound interest is

$$S - 24(2000) \approx 51{,}118.23 - 48{,}000$$ (3)
$$= 3{,}118.23 \text{ SR.}$$

Q5.

(a) In how many ways can four men and three women line up for a group picture? In how many ways can they line up if a woman is to be at each end? [8 points]

4 men + 3 women = 7 persons

– ${}_7P_7 = 7! = 5040$ ways. (4)

– The left end can be occupied by any of the 3 women, the right end by any of the remaining 2 women. Now the last woman and the 4 men can be arranged in the 5 middle positions in ${}_5P_5 = 5! = 120$ ways. Thus we have $(3)(2)(120) = 720$ ways. (4)

(b) If two ... le dice are tossed and the numbers on the top faces are noted. In how many ways can the sum be less than 6? List these possibilities [7 points]

(1, 1), (1, 2), (1, 3), (1, 4)
(2, 1), (2, 2), (2, 3) (3)
(3, 1), (3, 2)
(4, 1).
There are 10 possibilities. (4)

(c) An artist has created 15 original paintings, and she will exhibit some of them in two galleries. Four paintings will be sent to gallery I and nine to gallery II. In how many ways can this be done? [Note that: (15)(13)(11)(7)(5) = 75075] [10 points]

$n = 15$, $n_1 = 4$, $n_2 = 9$, $n_3 = 2$.

Thus we have

$$\frac{15!}{4!\,9!\,2!} = \frac{(15)(14)(13)(12)(11)(10)\,9!}{(4)(3)(2)\,9!\,2}$$

$$= (15)(13)(11)(7)(5)$$

$$= 75075 \text{ ways.}$$