

11.5 The Product and Quotient Rules

Rule 5. If $f(x)$ and $g(x)$ are differentiable functions then their product $f(x)g(x)$ is also differentiable. Moreover,

$$\frac{d\{f(x)g(x)\}}{dx} = \frac{d\{f(x)\}}{dx}g(x) + f(x)\frac{d\{g(x)\}}{dx} = f'(x)g(x) + f(x)g'(x).$$

AP 1(p. 566). Applying the Product Rule.

A taco stand sells 225 tacos per day at \$2 each. A business student's research tells him that for every \$0.15 decrease in the price, the stand will sell 20 more tacos per day. The revenue function for the taco stand is

$R(x) = (2 - 0.15x)(225 + 20x)$, where x is the number of \$0.15 reductions

in price. Find $\frac{dR}{dx}$.

Soln. Note that all the preliminary information is not needed to solve the problem. By the Product Rule

$$\begin{aligned}\frac{dR}{dx} &= (-0.15)(225 + 20x) + (2 - 0.15x)(20). \\ &= -33.75 - 3x + 40 - 3x \\ &= 6.25 - 6x.\end{aligned}$$

Example 3*(p.566) Differentiating a Product of Three Factors.

If $y = x(x - 2)(x + 2)$, find y' .

Soln. first note that, in general we have

$$\begin{aligned}(fgh)' &= [(fg)h]' = (fg)'h + (fg)h' = (f'g + fg')h + (fg)h' \\ &= f'gh + fg'h + fgh'\end{aligned}$$

Thus, by the Product Rule for three functions we see that

$$\begin{aligned}y' &= (1)(x - 2)(x + 2) + x(1)(x + 2) + x(x - 2)(1). \\ &= (x^2 - 4) + (x^2 + 2x) + (x^2 - 2x). \\ &= 3x^2 - 4.\end{aligned}$$

Rule 6. If $f(x)$ and $g(x)$ are differentiable functions then their quotient $f(x)/g(x)$ is also differentiable. Moreover,

$$\frac{d\{f(x)/g(x)\}}{dx} = \frac{\frac{d\{f(x)\}}{dx}g(x) - f(x)\frac{d\{g(x)\}}{dx}}{\{g(x)\}^2} = \frac{f'(x)g(x) - f(x)g'(x)}{\{g(x)\}^2}.$$

Example 5(p.567) Applying the Quotient Rule.

If $F(x) = \frac{4x^2 + 3}{2x - 1}$, find $F'(x)$.

Soln. By the Quotient Rule we see that

$$\begin{aligned} F'(x) &= \frac{\frac{d\{4x^2 + 3\}}{dx}(2x - 1) - (4x^2 + 3)\frac{d\{2x - 1\}}{dx}}{(2x - 1)^2} = \frac{8x(2x - 1) - (4x^2 + 3)2}{(2x - 1)^2}. \\ &= \frac{8x^2 - 8x - 6}{(2x - 1)^2} = \frac{2(4x^2 - 4x - 3)}{(2x - 1)^2} = \frac{2(2x + 1)(2x - 3)}{(2x - 1)^2}. \end{aligned}$$