

o81 Problem 10 version 1

$$\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{3 \tan^2 \theta}$$

$$\begin{aligned} \frac{\cos \theta - 1}{3 \tan^2 \theta} &= \frac{\cos \theta - 1}{3 \frac{\sin^2 \theta}{\cos^2 \theta}} \\ &= \frac{(\cos \theta - 1) \cos^2 \theta}{3(1 - \cos^2 \theta)} \\ &= \frac{(\cos \theta - 1) \cos^2 \theta}{3(1 - \cos \theta)(1 + \cos \theta)} \\ &= \frac{-\cos^2 \theta}{3(1 + \cos \theta)} \end{aligned}$$

$$\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{3 \tan^2 \theta} = \lim_{\theta \rightarrow 0} \frac{-\cos^2 \theta}{3(1 + \cos \theta)} = \frac{-1}{6}$$