

King Fahd University of Petroleum & Minerals
 Department of Mathematical Sciences
 MATH - 260 Semester 082 Quiz - 2

Name: _____ ID# _____ Section# _____

Problem. Solve the equation

$$3(1+x^2) \frac{dy}{dx} = 2xy(y^3-1), \quad (\Rightarrow)$$

$$3(1+x^2) \frac{dy}{dx} = 2xy^4 - 2xy \quad (\Leftarrow)$$

$$\frac{dy}{dx} + \frac{2x}{3(1+x^2)} y = 2xy^4$$

This is a Bernoulli equation. We do the substitution $u = y^{-3} = y^{-3}$. The equation

becomes

$$\frac{du}{dx} - 3 \cdot \frac{2x}{3(1+x^2)} u = -3(2x) \quad (\Rightarrow)$$

$$\frac{du}{dx} - \frac{2x}{1+x^2} u = -6x \quad (\text{linear})$$

$$\Rightarrow \frac{d}{dx} \left[u \cdot e^{-\int \frac{2x}{1+x^2} dx} \right] = -6x e^{-\ln|1+x^2|} \quad (\Rightarrow)$$

$$\frac{d}{dx} \left[u \cdot e^{-\ln|1+x^2|} \right] = -6x e^{-\ln|1+x^2|} \quad (\Rightarrow)$$

$$\frac{u}{1+x^2} = \frac{-6x}{1+x^2} + C \quad (\Rightarrow)$$

$$u = -6x + C(1+x^2) \quad (\Rightarrow)$$

$$y^{-3} = -6x + C(1+x^2) \quad (\Rightarrow)$$

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Problem. Solve the equation

$$x \frac{dy}{dx} - (1+x)y = xy^2, \quad (\Leftrightarrow)$$

$$\frac{dy}{dx} - \left(\frac{1+x}{x}\right)y = y^2, \quad \text{Bernoulli with } n=2$$

We do the substitution

$$u = y^{1-2} = y^{-1}. \quad \text{The equation}$$

becomes

$$\frac{du}{dx} + \left(\frac{1}{x} + 1\right)u = -1 \quad (\text{linear})$$

$$\Leftrightarrow \frac{d}{dx} \left[u \cdot e^{\int \left(\frac{1}{x} + 1\right) dx} \right] = -e^{\int \left(\frac{1}{x} + 1\right) dx}$$

$$\frac{d}{dx} \left[u \cdot e^{+\ln|x|+x} \right] = -e^{+\ln|x|+x}$$

$$\Rightarrow u \cdot x e^{+x} = - \int x e^{+x} dx + c \quad \text{assume } x > 0$$

We have

$$u = \frac{1}{x e^{+x}} \left[- \int x e^x dx + c \right]$$

$$= \frac{1}{x e^{+x}} (e^x - x e^x + c)$$

$y = \frac{x e^x}{e^x - x e^x + c}$