**King Fahd University of Petroleum & Minerals**
Department of Mathematical Sciences
MATH - 260 Semester 082 **Quiz - 1**

Name: _____ ID#_____ Section#_____

**Problem 1.** Solve the initial value problem

$$2\sqrt{x}\,\frac{dy}{dx} = \cos^2 y, \qquad y(4) = \frac{\pi}{4}$$

$\frac{dy}{\cos^2 y} = \frac{dx}{2\sqrt{x}} \Rightarrow \tan y = \sqrt{x} + C$

$x = 4, \; y = \frac{\pi}{4} \Rightarrow 1 = 2 + C \Rightarrow C = -1$

$\tan y = \sqrt{x} - 1 \Rightarrow y(x) = \tan^{-1}(\sqrt{x} - 1)$

**Problem 2.** A meat roast, initially at 50 °$F$, is placed in a 375 °$F$ at 5:00 pm. After 75 minutes it is found that the temperature $T(t)$ of the roast is 125°$F$. When will the roast be 150°$F$? Let $T(t)$ be the temperature of the roast at time $t$.

By Newton's Law

$\frac{dT}{dt} = +k(375 - T)$ $\quad k > 0$

$\Rightarrow \int \frac{dT}{375 - T} = \int k\,dt \Rightarrow -\ln(375 - T) = kt + C$

$\Rightarrow T(t) = 375 + Ae^{-kt}$ $\quad T(0) = 50$

$\Rightarrow A = -325$ and $T(t) = 375 - 325e^{-kt}$

$T(75) = 125 \Rightarrow k = -\frac{1}{75}\ln\left(\frac{250}{325}\right) \approx 0.0035$

Now we need to find $t$ such that $T(t) = 150 \Rightarrow 150 = 375 - 325e^{-0.0035t} \Rightarrow$

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**Problem 1.** Solve the initial value problem

$$x\frac{dy}{dx} - y = 2x^2y, \qquad y(1) = 1$$

$x\frac{dy}{dx} = 2x^2y + y \iff \frac{dy}{dx} = \left(\frac{2x^2+1}{x}\right)y$ (Separable)

$\Rightarrow \frac{dy}{y} = \left(2x + \frac{1}{x}\right)dx \Rightarrow \ln|y| = x^2 + \ln|x| + C$

$\Rightarrow y = A\,e^{x^2+\ln|x|}$ $\quad y(1) = 1 \Rightarrow 1 = Ae^1 \Rightarrow$

$A = e^{-1}$. Particular solution is $y(x) = e^{-1}\cdot e^{x^2+\ln|x|} = e^{x^2+\ln|x|-1}$

**Problem 2.** A Pitcher of buttermilk initially at 25 °C is to be cooled in freezer where the tenperature is 0°C. Suppose that the temperature of the buttermilk has dropped to 15°C after 20 minutes. When will it be at 5°C?

Let $T$ be the temperature of the Pitcher at time $t$.

According to Newton's Law

$\frac{dT}{dt} = -k(T-0) \quad (k>0) \iff$

$\frac{dT}{dt} = -kT \iff \frac{dT}{T} = -k\,dt \Rightarrow \ln|T| = -kt + C$

$\Rightarrow T = Ae^{-kt}$. $T(0) = 25 \Rightarrow A = 25$ and

$T(t) = 25e^{-kt}$. $T(20) = 15 \iff 25e^{-20k} = 15$,

$\iff e^{-20k} = \frac{15}{25} = \frac{3}{5} \Rightarrow -20k = \ln\frac{3}{5}$ &

$k = -\frac{1}{20}\ln\left(\frac{3}{5}\right)$. $T(t) = 25e^{\frac{1}{20}\ln(3/5)t}$

Now we need to find $t$ such that $T(t) = 5$. i.e.

$\boxed{\quad}$