KFUPM SEM I Term 051 Name: ______ Serial #: ______

MATH 260-2-4 Quiz # 1 ID: #. KEY

1 (5-points) Solve the initial value problem

$$\frac{dy}{dx} = \frac{2xy + 3x^2 + 3}{x^2 + 1}, \quad y(1) = \frac{7\pi}{2}.$$

$\Rightarrow (x^2+1)\frac{dy}{dx} - 2xy = 3(x^2+1)$

which is a first-order linear DE $\Rightarrow$

$$\frac{dy}{dx} - \frac{2x}{x^2+1} y = 3 \Rightarrow$$

$$\text{I.F} = e^{-\int \frac{2x}{x^2+1} dx} = e^{-\ln(x^2+1)} = \frac{1}{x^2+1}$$

$\Rightarrow$ The G.S :

$$\frac{1}{x^2+1} y = \int \frac{3}{x^2+1} dx + C$$

$$= 3\tan^{-1} x + C \Rightarrow$$

G.S. is $y = 3(x^2+1)\tan^{-1}x + c(x^2+1)$

$y(1) = \frac{7\pi}{2} \Rightarrow \frac{7\pi}{2} = 6\left(\frac{\pi}{4}\right) + 2C$

$\Rightarrow 2\pi = 2C \Rightarrow \boxed{C = \pi}$

$\Rightarrow$ The required solution is

$$y = 3(x^2+1)\tan^{-1}x + \pi(x^2+1).$$

Quiz 1 (051)

2. (5-points) Show the differential equation

$$xy\frac{dy}{dx} - (x+y)^2 = 0$$

is homogeneous, then find its general solution.

$\Rightarrow (x^2+2xy+y^2)\,dx - xy\,dy = 0$ ①

$\Rightarrow M(x,y) = x^2+2xy+y^2$ is homogeneous of degree 2

& $N(x,y) = -xy$ is also homogeneous of degree 2

$\Rightarrow$ the Given DE is homogeneous.

Let $y = vx \Rightarrow dy = v\,dx + x\,dv$ ②

① & ② $\Rightarrow (x^2 + 2x^2v + v^2x^2)\,dx - x^2v\,(v\,dx + x\,dv) = 0$

$\Rightarrow x^2(1+2v+v^2-v^2]\,dx - x^3v\,dv = 0$

$\Rightarrow x^2(1+2v)\,dx - x^3v\,dv = 0$

$\Rightarrow \frac{1}{x}\,dx - \frac{v}{1+2v}\,dv = 0$

$\Rightarrow \frac{1}{x}\,dx - \frac{1}{2}\left(1 - \frac{1}{1+2v}\right)dv = 0$

$\Rightarrow \int \frac{1}{x}\,dx - \frac{1}{2}\int\left(1 - \frac{1}{1+2v}\right)dv = 0$

$\Rightarrow \ln|x| - \frac{1}{2}v + \frac{1}{4}\ln|1+2v| = C$

$\Rightarrow$ The G.S. is

$$\ln|x| - \frac{1}{2}\left(\frac{y}{x}\right) + \frac{1}{4}\ln\left(1+\frac{2y}{x}\right) = C.$$