KFUPM SEM II (Term 062) Name: ______ Serial #: ______

MATH 102 Quiz # 5 ID: #: KEY Section #: ______

1. (3-points) Find a formula for the general term a_n of the sequence

$$\left\{\frac{3}{5}, +\frac{6}{7}, \frac{9}{9}, +\frac{12}{11}, \frac{15}{13}, \dots\right\}.$$

Then determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{3n}{2n+3}, \quad n \geq 1$$

$$\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3n}{2n+3}$$

$$= \lim_{n\to\infty} \frac{3}{2+\frac{3}{n}} = \frac{3}{2}$$

$\Rightarrow$ The sequence converges to $\frac{3}{2}$.

2. (4-points) Find the values of x for which the series $\displaystyle\sum_{n=3}^{\infty} \frac{(2x+1)^n}{3^{n-1}}$ converges. Then, find the sum for those values of x.

The series is

$$\frac{(2x+1)^3}{3^2} + \frac{(2x+1)^4}{3^3} + \frac{(2x+1)^5}{3^4} + \cdots$$

which is a geometric series with first term $a = \frac{(2x+1)^3}{3^2}$ and common ratio r is $r = \frac{2x+1}{3}$.

Therefore, the series is convergent if $|r| < 1 \Rightarrow \left|\frac{2x+1}{3}\right| < 1$

$\Rightarrow -3 < 2x+1 < 3 \Rightarrow$

$-4 < 2x < 2 \Rightarrow -2 < x < 1$

and for those values of x the series has the sum $= \frac{a}{1-r}$

$$= \frac{\frac{(2x+1)^3}{3^2}}{1 - \frac{2x+1}{3}} = \frac{(2x+1)^3}{3(2-2x)}.$$

$$= \frac{(2x+1)^3}{6(1-x)}, \quad -2 < x < 1.$$

P.T.O. $\rightarrow$

3. (4-points) Show that the integral test can be used to test the series $\sum_{n=1}^{\infty} \frac{4}{n^4}$ for convergence or divergence. If it converges, then find an upper bound for the size of the error if its sum S is approximated by S_{50} (write your answer in a decimal form).

Let $f(n) = \frac{4}{n^4}$, $n \geq 1 \Rightarrow$

$f(x) = \frac{4}{x^3}$, $x \geq 1$ which in a continuous and positive function for all $x \geq 1$.

$$f'(x) = -\frac{12}{x^4} < 0 \text{ for all } x \geq 1$$

$\Rightarrow$ f in decreasing for all $x \geq 1$

$\Rightarrow$ The integral test can be used

$$\int_1^{\infty} \frac{4}{x^4}\,dx = \lim_{t\to\infty} \int_1^t \frac{4}{x^4}\,dx$$

$$= \lim_{t\to\infty} \left[-\frac{4}{3x^3}\right]_1^t$$

$$= \lim_{t\to\infty} \left[\frac{-4}{3t^2} + \frac{4}{3}\right] = \frac{4}{3}$$

$\Rightarrow$ The series in convergent

$$R_{50} \leq \int_{50}^{\infty} \frac{4}{x^4}\,dx$$

$$= \lim_{t\to\infty} \left[\frac{-4}{3x^3}\right]_{50}^t = \frac{4}{3} \frac{1}{(5^3)(10)^3}$$

$$= \left(\frac{4}{3}\right)(0.2)^3 (0.001)$$

$$\approx (1.3)(0.008)(0.001)$$

$$= 0.0000104.$$

4. (4-points) Find a closed form for the general form S_n of the sequence of partial sums of the series $\sum \frac{1}{\cancel{n^2+8n+15}}$ $n^2+7n+12$. Then find, if possible, the sum of the series.

$$\frac{1}{n^2+7n+12} = \frac{1}{(n+3)(n+4)}$$

$$= \frac{1}{n+3} - \frac{1}{n+4} \Longrightarrow$$

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

$$= \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \left(\frac{1}{6} - \frac{1}{7}\right) + \cdots + \left(\frac{1}{n+2} - \frac{1}{n+3}\right) + \left(\frac{1}{n+3} - \frac{1}{n+4}\right)$$

$$= \frac{1}{4} - \frac{1}{n+4}$$

(A telescoping Sum) $\Longrightarrow$

Therefore, $\frac{1}{4} - \frac{1}{n+4}$ in the closed form for S_n.

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(\frac{1}{4} - \frac{1}{n+4}\right)$$

$$= \frac{1}{4}$$

$\Rightarrow$ The series in Convergent and has the sum $\frac{1}{4}$.