

KFUPM SEM II (Term 062) Name: _____ Serial #: _____

MATH 102 Quiz # 4 ID: # KEY Section #: _____1. (4-points) Evaluate $\int \frac{dx}{(4+x^2)^{3/2}} = I$

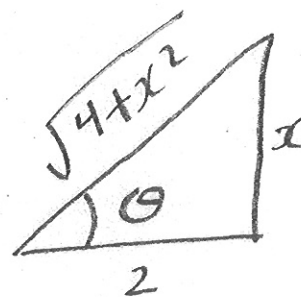
$$\text{Let } x = 2 \tan \theta \Rightarrow (4+x^2)^{3/2} = (4 \sec^2 \theta)^{3/2} = 8 \sec^3 \theta,$$

$$\text{and } dx = 2 \sec^2 \theta d\theta$$

$$\Rightarrow I = \int \frac{2 \sec^2 \theta d\theta}{8 \sec^3 \theta} = \frac{1}{4} \int \frac{1}{\sec \theta} d\theta$$

$$= \frac{1}{4} \int \cos \theta d\theta = \frac{1}{4} \sin \theta + C$$

$$= \frac{1}{4} \frac{x}{\sqrt{4+x^2}} + C$$

2. (4-points) Test for convergence or divergence the improper integral $\int_0^1 \frac{dx}{\sqrt{x}(x+1)} = I$

$$I = \lim_{t \rightarrow 0^+} \int_t^1 \frac{dx}{\sqrt{x}(x+1)}$$

$$\frac{\sqrt{2}}{\sqrt{2}} \quad \frac{x_0}{2} \quad \frac{y_0}{2}$$

to find $\int \frac{dx}{\sqrt{x}(x+1)}$, Let $u^2 = x \Rightarrow dx = 2u du$

$$\Rightarrow \int \frac{dx}{\sqrt{x}(x+1)} = \int \frac{2u du}{u(u^2+1)} = \int \frac{2 du}{u^2+1} = \tan^{-1} \sqrt{x} + C$$

$$\Rightarrow I = \lim_{t \rightarrow 0^+} \left[\tan^{-1} \sqrt{x} \right]_t^1 = \lim_{t \rightarrow 0^+} \left[\tan^{-1} 1 - \tan^{-1} \sqrt{t} \right]$$

$$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

$$\Rightarrow \text{the integral converges and has the value } \frac{\pi}{4}.$$

3. (7-points) Evaluate $\int \frac{5x+3}{x^2(x^2+1)} dx = I$

$$\frac{5x+3}{x^2(x^2+1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{Cx+D}{x^2+1}$$

$$\Rightarrow 5x+3 = A(x^2+1) + Bx(x^2+1) + (Cx+D)x^2$$

$$x=0 \Rightarrow \boxed{3 = A}$$

$$\text{Coef } x^2 \mid 0 = A + D \Rightarrow \boxed{D = -3}$$

$$\text{Coef } x \mid \boxed{5 = B}$$

$$\text{Coef } x^3 \mid 0 = B + C \Rightarrow \boxed{C = -5}$$

$$\Rightarrow I = \int \left[\frac{3}{x^2} + \frac{5}{x} + \frac{-5x-3}{x^2+1} \right] dx$$

$$= 3 \int \frac{1}{x^2} dx + 5 \int \frac{1}{x} dx - 5 \int \frac{x}{x^2+1} dx - 3 \int \frac{dx}{x^2+1}$$

$$= -\frac{3}{x} + 5 \ln|x| - \frac{5}{2} \ln(x^2+1) - 3 \tan^{-1} x + C.$$