KFUPM	SEM II (Term 062)	Name:	Serial #:
MATH 102	Quiz # 6	ID: #: KEY	Section #:

1. (4-points) Use the comparison test to determine whether the series $\sum_{n=1}^{\infty} \frac{5+2n}{(1+n^2)^2}$ converges or diverges.

$$a_n = \frac{5+2n}{(1+n^2)^2} < \frac{5+2n}{n^4} = \frac{5}{n^4} + \frac{2}{n^3} \text{ for all } n \geq 1$$

But $\sum_{n=1}^{\infty} \frac{5}{n^4} = 5 \sum_{n=1}^{\infty} \frac{1}{n^4}$ and $\sum_{n=1}^{\infty} \frac{2}{n^3} = 2 \sum_{n=1}^{\infty} \frac{1}{n^3}$

are both convergent p-series ($p=4$ and $p=3$).

Therefore, the given series is convergent by the comparison test.

2. (4-points) Use the limit comparison test to determine whether the series $\sum_{n=1}^{\infty} \frac{1+n+4n^2}{\sqrt{1+3n^2+9n^6}}$ converges or diverges.

Let $a_n = \frac{1+n+4n^2}{\sqrt{1+3n^2+9n^6}}$ and $b_n = \frac{1}{n}$ $\Rightarrow$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n+n^2+4n^3}{\sqrt{1+3n^2+9n^6}} = \lim_{n\to\infty} \frac{\frac{1}{n^2}+\frac{1}{n}+4}{\sqrt{\frac{1}{n^6}+\frac{3}{n^4}+9}}$$

$$= \frac{4}{\sqrt{9}} = \frac{4}{3} \text{ which is finite and } \neq 0$$

Therefore, the given series is divergent by the limiting comparison test since $\sum_{n=1}^{\infty} \frac{1}{n}$ is a divergent series.

3. (4-points) Use the alternating series test to determine whether the series $\sum_{n=1}^{\infty}(-1)^{n+1}\, ne^{-\frac{n}{3}}$ converges or divergence.

$$b_n = ne^{-\frac{n}{3}} > 0. \text{ Let } f(x) = x\, e^{-\frac{x}{3}} \Rightarrow$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x}{e^{x/3}} = \lim_{x\to\infty} \frac{1}{\frac{1}{3}e^{x/3}} = 0$$

$$\text{and } f'(x) = e^{-x/3} - \frac{1}{3}x e^{-\frac{x}{3}} = \frac{1}{3}e^{-x/3}(3-x)$$

$\Rightarrow f'(x) < 0$ for $x > 3 \Rightarrow \{b_n\}$ is eventually decreasing.

Therefor the series is convergent by the alternating series test.

4. (4-points) Test the series $\sum_{n=1}^{\infty}(-1)^n \frac{(2n+1)^5}{4^n}$ for absolute convergence.

$$a_n = (-1)^n \frac{(2n+1)^5}{4^n} \Rightarrow a_{n+1} = (-1)^{n+1} \frac{(2n+3)^5}{4^{n+1}}$$

$$\Rightarrow \left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(2n+3)^5}{4^{n+1}} \cdot \frac{4^n}{(2n+1)^5}\right| = \frac{1}{4}\left(\frac{2+3/n}{2+\frac{1}{n}}\right)^5$$

$\Rightarrow \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{4} < 1 \Rightarrow$ The series is absolutely convergent according to the ratio test.