KFUPM SEM II (Term 062) Name: KEY Serial #: ____

MATH 102 Quiz # 2 ID: #: ____ Section #: ____

1. (3-points) Find $\int \frac{\tan^{-1} 3x}{1+9x^2}\,dx. = I$

Let $u = \tan^{-1} 3x \Rightarrow du = \frac{3}{1+9x^2}\,dx$

$$\Rightarrow I = \frac{1}{3}\int u\,du = \frac{1}{6}u^2 + C$$

$$= \frac{1}{6}(\tan^{-1}3x)^2 + C$$

2. (2-points) Find $\int \frac{2+5x}{\sqrt{1+4x+5x^2}}\,dx. = I$

Let $u = 1+4x+5x^2 \Rightarrow du = (4+10x)\,dx$

$\Rightarrow \frac{1}{2}du = (2+5x)\,dx \Rightarrow$

$$I = \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int u^{-1/2}\,du = u^{1/2} + C$$

$$= \sqrt{1+4x+5x^2} + C.$$

3. (3-points) Evaluate $\int_0^1 \sqrt[3]{x^4-1}\,x^7\,dx. = \int_0^1 \sqrt[3]{x^4-1}\;x^4(x^3dx) = I$

Let $u = x^4-1 \Rightarrow du = 4x^3dx$ & $x^4 = u+1$

Also when $x=0 \Rightarrow u=-1$, and when $x=1 \Rightarrow u=0$

$$\Rightarrow I = \int_{-1}^{0} u^{1/3}(u+1)\left(\frac{1}{4}du\right) = \frac{1}{4}\int_{-1}^{0}\left(u^{4/3}+u^{1/3}\right)du$$

$$= \frac{1}{4}\left[\frac{3}{7}u^{7/3} + \frac{3}{4}u^{4/3}\right]_{-1}^{0}$$

$$= \frac{1}{4}\left[0 - \left(-\frac{3}{7}+\frac{3}{4}\right)\right] = -\frac{3}{4}\left(\frac{3}{28}\right) = -\frac{9}{112}.$$

4. (a) (3-points) Sketch the region bounded by the parabola $x = 2y - y^2$ and the line $x = -2y$.

$x = 2y - y^2 = -(y-1)^2 + 1$

vertex at $(1,1)$

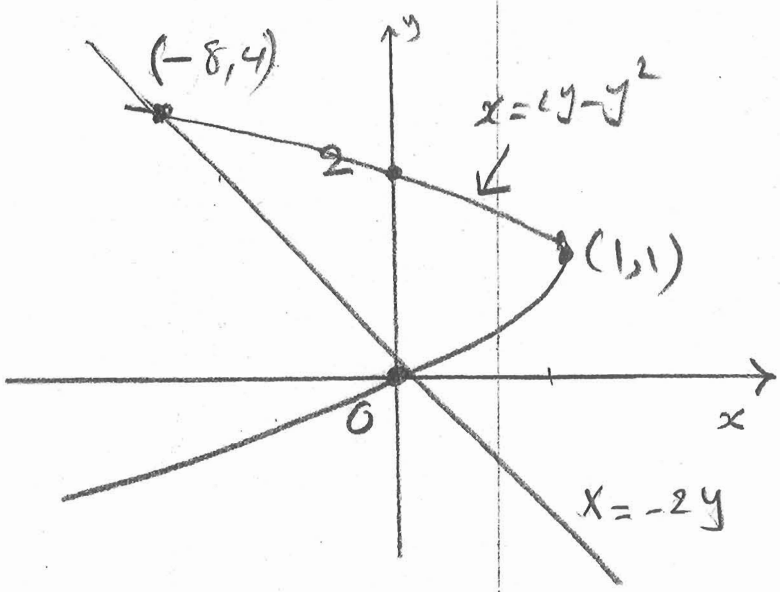
pts of intersection

$-2y = 2y - y^2 \Rightarrow$

$y^2 - 4y = 0 = y(y-4)$

$y = 0 \;,\; y = 4 \Rightarrow$

$(0,0)\,,\,(-8,4)$



(b) (2-points) Set up, **but do not evaluate**, an integral for the area of the region in part (a).

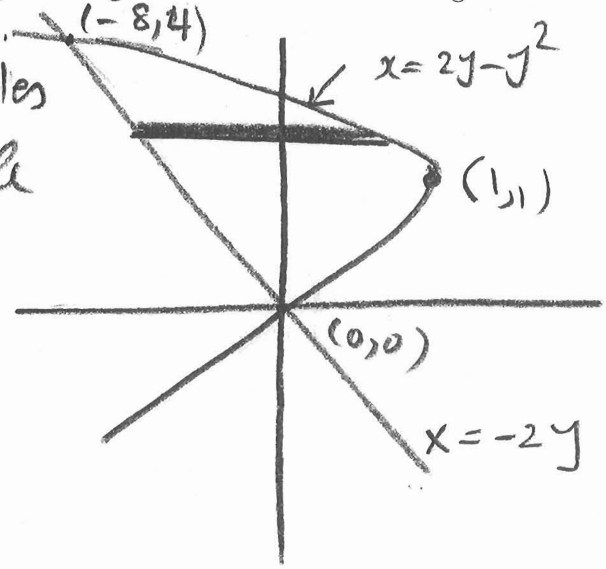
use horizontal typical rectangles

The area of a typical rectangle

$= [(2y - y^2) - (-2y)]\,dy$

$= (4y - y^2)\,dy \Rightarrow$

The area $= \displaystyle\int_0^4 (4y - y^2)\,dy$



(c) (2-points) Use the disk/washer method to set up an integral, **but do not evaluate**, for the volume of the solid obtained by rotating the region in part (a) about the line $x = 4$.

use washers:

The volume of a typical washer

$= \pi\left[(4 + 2y)^2 - (4 - 2y + y^2)^2\right] dy$

The required volume

$= \pi \displaystyle\int_0^4 \left[(4 + 2y)^2 - (4 - 2y + y^2)^2\right] dy.$