KFUPM	SEM II (Term 062)	Name: KEY	Serial #:
MATH 102	Quiz # 2	ID: #:	Section #:

1. (3-points) Find $\int \frac{\tan^{-1}\frac{1}{3}x}{9+x^2}\,dx. = I$

Let $u = \tan^{-1}\frac{1}{3}x \Rightarrow du = \frac{1/3}{1+\frac{1}{9}x^2}\,dx = \frac{3\,dx}{9+x^2}$

$I = \frac{1}{3}\int u\,du = \frac{1}{6}u^2 + C$

$= \frac{1}{6}\left(\tan^{-1}\frac{1}{3}x\right)^2 + C$

2. (2-points) Find $\int \frac{1+5x}{\sqrt{1-6x-15x^2}}\,dx. = I$

Let $u = 1-6x-15x^2 \Rightarrow du = (-6-30x)\,dx \Rightarrow$

$-\frac{1}{6}du = (1+5x)\,dx \Rightarrow$

$I = -\frac{1}{6}\int \frac{1}{\sqrt{u}}\,du = -\frac{1}{6}\int u^{-1/2}\,du = -\frac{1}{3}u^{1/2} + C$

$= -\frac{1}{3}\sqrt{1-6x-15x^2} + C.$

3. (3-points) Evaluate $\int_{-1}^{0} \sqrt[3]{x^5+1}\,x^9\,dx. = \int_{-1}^{0} \sqrt[3]{x^5+1}\;x^5\,(x^4\,dx) = I$

Let $u = x^5+1 \Rightarrow du = 5x^4\,dx$ and $x^5 = u-1$

Also, when $x=-1 \Rightarrow u=0$, and when $x=0 \Rightarrow u=1$

$\Rightarrow I = \int_0^1 u^{1/3}(u-1)\left(\frac{1}{5}du\right) = \frac{1}{5}\int_0^1 (u^{4/3} - u^{1/3})\,du$

$= \frac{1}{5}\left[\frac{3}{7}u^{7/3} - \frac{3}{4}u^{4/3}\right]_0^1 = \frac{1}{5}\left[\left(\frac{3}{7} - \frac{3}{4}\right) = \frac{3}{5}\left(-\frac{3}{28}\right)\right.$

$= -\frac{9}{140}.$

4. (a) (3-points) Sketch the region bounded by the parabola $x = y^2 - 2y$ and the line $x = 2y$.

$x = y^2 - 2y = (y-1)^2 - 1$

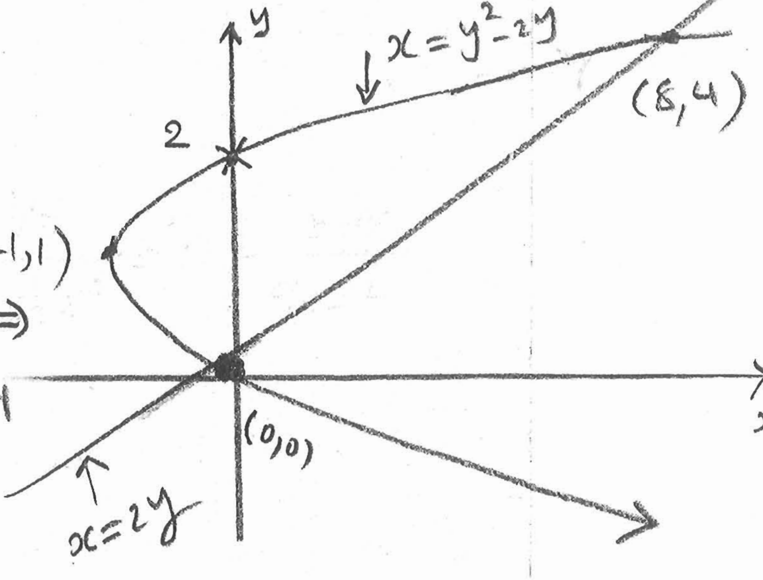
Vertex at $(-1, 1)$

Pts of intersection

$2y = y^2 - 2y \Rightarrow y^2 - 4y = 0 \Rightarrow$

$y(y-4) = 0 \Longrightarrow y = 0,\ y = 4$

Pts: $(0,0)$, $(8,4)$



(b) (2-points) Set up, **but do not evaluate**, an integral for the area of the region in part (a).

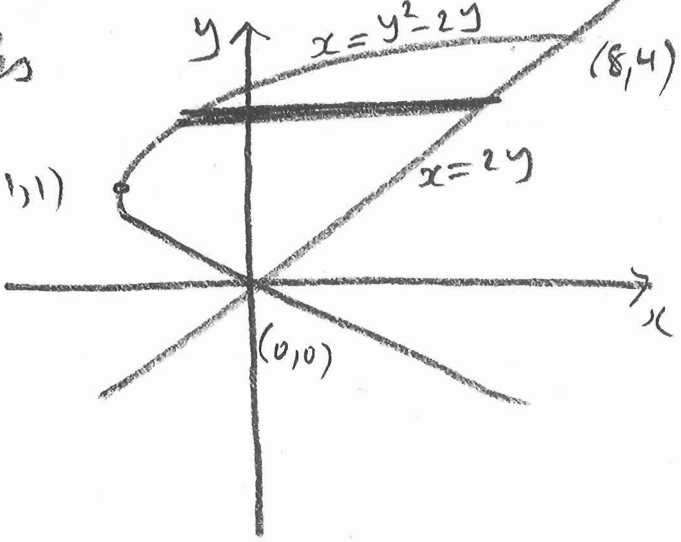
use horizontal typical rectangles

The area of a typical rectangle

$= [2y - (y^2 - 2y)]\,dy$

$= (4y - y^2)\,dy$

The area $= \displaystyle\int_0^4 (4y - y^2)\,dy$



(c) (2-points) Use the disk/washer method to set up an integral, **but do not evaluate**, for the volume of the solid obtained by rotating the region in part (a) about the line $x = -4$.

Use Washers:

The volume of a typical washer

$= \pi\left[(2y+4)^2 - (y^2 - 2y + 4)^2\right] dy$

$\Longrightarrow$ The required volume

$= \displaystyle\int_0^4 \pi\left[(2y+4)^2 - (y^2 - 2y + 4)^2\right] dy$