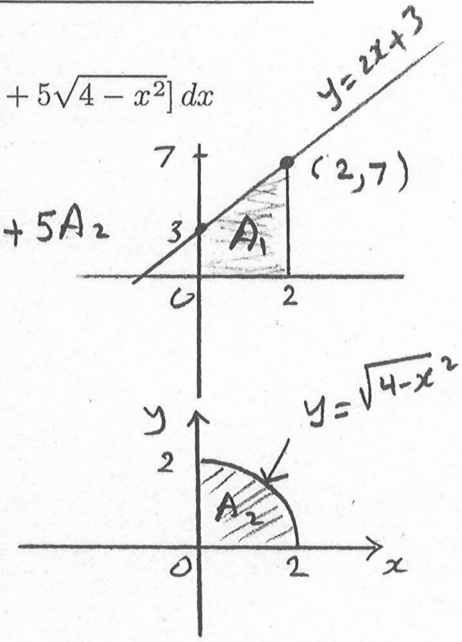
KFUPM | SEM II (Term 062) | Name: KEYS | Serial #: ______

MATH 102 | Quiz # 1 | ID: #: ______ | Section #: ______

1. (4-points) Find the exact value of the integral $\int_0^2 [(2x+3) + 5\sqrt{4-x^2}]\,dx$ by interpolating it in terms of area.



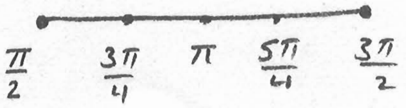
$$I = \int_0^2 (2x+3)\,dx + 5\int_0^2 \sqrt{4-x^2}\,dx = A_1 + 5A_2$$

$$A_1 = \frac{1}{2}(2)(3+7) = 10$$

$$A_2 = \frac{1}{4}(\pi 4) = \pi$$

$$\Rightarrow \quad I = 10 + 5\pi.$$

2. (3-points) Evaluate the Riemann sum for $f(x) = \cos x$, $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ with four subintervals, taking the sample points to be the right endpoints.



The required Riemann sum =

$$\frac{\pi}{4}\left[\cos\frac{3\pi}{4} + \cos\pi + \cos\frac{5\pi}{4} + \cos\frac{3\pi}{2}\right]$$

$$= \frac{\pi}{4}\left[-\frac{\sqrt{2}}{2} - 1 - \frac{\sqrt{2}}{2} + 0\right] = -\frac{\pi}{4}(\sqrt{2}+1).$$

3. (2-points) Express $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^{5/2} + \left(\frac{2}{n}\right)^{5/2} + \cdots + \left(\frac{n}{n}\right)^{5/2}\right]$ as an integral over the interval $[0,1]$, then find its value.

We can take $f(x) = x^{5/2} \Longrightarrow$

$$\text{The given limit} = \int_0^1 x^{5/2}\,dx = \frac{2}{7}\left[x^{7/2}\right]_0^1$$

$$= \frac{2}{7}[1-0] = \frac{2}{7}.$$

4. (4-points) Express the integral $\int_0^2 (4x^2+3)\,dx$ as a limit of a sum over the interval $[0,2]$, then find the exact value of the limit.

$$I = \int_0^2 (4x^2+3)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(4x_i^{*2} + 3\right) \Delta x$$

$\Delta x = \frac{2}{n}$, $x_i^* = i\left(\frac{2}{n}\right)$ (right end point) $\quad 0 \quad \frac{2}{n} \quad 2\left(\frac{2}{n}\right) \quad 3\left(\frac{2}{n}\right) \quad 2$

$$I = \lim_{n\to\infty} \sum_{i=1}^{n} \left[4\left(\frac{4i^2}{n^2}\right) + 3\right]\left(\frac{2}{n}\right)$$

$$I = \lim_{n\to\infty} \left(\frac{32}{n^3} \sum_{i=1}^{n} i^2\right) + \lim_{n\to\infty} \left(\frac{6}{n} \sum_{i=1}^{n} 1\right)$$

$$= \lim_{n\to\infty} \frac{32}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{6}{n}(n)$$

$$= \lim_{n\to\infty} \frac{16}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + 6 = \frac{32}{3} + 6 = \frac{50}{3}.$$

5. (2-points) Show that $\int_0^{\pi/4} 8x\cos x\,dx \le \frac{\pi^2}{4}$.

$8x\cos x \le 8x$ because $\cos x \le 1$

$$\Longrightarrow \int_0^{\pi/4} 8x\cos x\,dx \le \int_0^{\pi/4} 8x\,dx = \frac{8}{2}x^2\Big]_0^{\pi/4}$$

$$= 4\left(\frac{\pi^2}{16} - 0\right) = \frac{\pi^2}{4}.$$