Total 30 pts

Math 102-4-5 Quiz #3 (KEY) Summer (073)

① $y = \dfrac{\sin^2 3x}{(1-\cos 3x)^2} = \dfrac{(1-\cos 3x)(1+\cos 3x)}{(1-\cos 3x)^2} \Rightarrow$

1 pt $y = \dfrac{1+\cos 3x}{1-\cos 3x} \Rightarrow$

2 pts $\dfrac{dy}{dx} = \dfrac{(1-\cos 3x)(-3\sin x) - (1+\cos 3x)(3\sin 3x)}{(1-\cos 3x)^2}$

1 pt $= \dfrac{-3\sin x + \cancel{3\sin 3x\cos 3x} - 3\sin 3x - \cancel{3\cos 3x\sin 3x}}{(1-\cos 3x)^2}$

1 pt $= -6\sin 3x / (1-\cos 3x)^2$. # $\boxed{5 \text{ pts}}$

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② $\dfrac{d}{dx} \dfrac{\sqrt{(2x+3)^2+1}}{2x+3} =$

$$\frac{(2x+3)\cdot\dfrac{4(2x+3)}{2\sqrt{(2x+3)^2+1}} - \sqrt{(2x+3)^2+1}\cdot 2}{(2x+3)^2}$$ 2 pts

$$= \frac{\cancel{2(2x+3)^2} - \cancel{2(2x+3)^2} - 2}{(2x+3)^2\sqrt{(2x+3)^2+1}}$$ 2 pts

$$= \frac{-2}{(2x+3)^2\sqrt{(2x+3)^2+1}} .$$ # $\boxed{5 \text{ pts}}$ 1 pt

Quiz #3 (KEY) (Cont.) (073)

3) $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = 7 \Rightarrow f'(2) = 7$ (1pt)

and $g(x) = (x^2 - 7) f(x) \Rightarrow$

$g'(x) = (x^2-7) f'(x) + 2x f(x)$ (1pt)

$\Rightarrow g'(2) = (-3) \cdot f'(2) + 4f(2) = (-3)(7) + (4)(3)$

$\Rightarrow g'(2) = -9 =$ The slope of the tangent line to the curve $y = g(x)$ at $(2, g(2))$ (2pts)

$\Rightarrow$ The slope of the required normal line $= \frac{1}{9}$ (1pt)

Now $g(2) = (4-7) f(2) = (-3)(3) = -9$ (1pt)

$\Rightarrow$ slope of normal line $= \frac{1}{9}$, point $(2, -9)$

$\Rightarrow$ The required equation is

8pts $y + 9 = \frac{1}{9}(x - 2) \Rightarrow \boxed{x - 9y - 83 = 0}$ (2pts)

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4) (a) $S(t) = 2\sin t - t \Rightarrow v(t) = 2\cos t - 1$ (1pt)

$\Rightarrow v(t) = 0$ when $\cos t = \frac{1}{2} \Rightarrow v(t) = 0$ when $t = \frac{\pi}{3}$ and $t = \frac{5\pi}{3}$ on $[0, 2\pi]$. (1pt)

$\Rightarrow$ The total distance required $=$

$|S(\frac{\pi}{3}) - S(0)| + |S(\frac{5\pi}{3}) - S(\frac{\pi}{3})| + |S(2\pi) - S(\frac{5\pi}{3})|$

(1pt)

Quiz #3 (KEY) Cont.) (073) ③

$$= \left|\left(2\left(\frac{\sqrt{3}}{2}\right) - \frac{\pi}{3}\right) - 0\right| + \left|\left(2\frac{\sqrt{3}}{2} - \frac{5\pi}{3}\right) - \left(2\frac{\sqrt{3}}{2} - \frac{\pi}{3}\right)\right|$$

$$+ \left|(0 - 2\pi) - \left(2\frac{\sqrt{3}}{2} - \frac{5\pi}{3}\right)\right|$$ (1 pt)

$$= \left(\sqrt{3} - \cancel{\frac{\pi}{3}}\right) + \left(\frac{4\pi}{3}\right) + \left(\cancel{\frac{\pi}{3}} + \sqrt{3}\right)$$

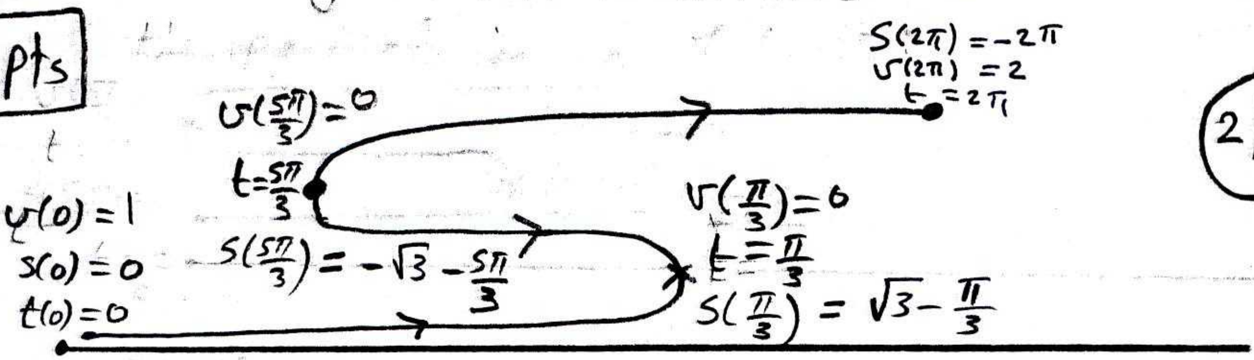
$$= \left(2\sqrt{3} + \frac{4\pi}{3}\right) \text{ meters.}$$

④ (b) First we draw a sign diagram of $v(t) = 2\cos t - 1$ on $[0, 2\pi]$ (1 pt)

$v(t)$	$+ + +$	$\frac{\pi}{3}$	$- \quad - \quad -$	$\frac{5\pi}{3}$	$+ + +$	2π

$\Rightarrow$ A diagram that illustrates the motion of

$\boxed{7 \text{ pts}}$



(2 pts) #

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⑤ $$\lim_{x\to 0} \frac{5x^2 - \tan^2 2x}{1 - \cos 3x} = \lim_{x\to 0} \frac{5x^2 - \tan^2 2x}{2\sin^2 \frac{3x}{2}}$$ (1 pt)

$$= \lim_{x\to 0}\left[\frac{5x^2}{2\sin^2\frac{3x}{2}} - \frac{\tan^2 2x}{2\sin^2\frac{3x}{2}}\right]$$ (2 pts)

$$= \lim_{x\to 0}\left[\frac{5}{2}\left(\frac{1}{\frac{\sin\frac{3x}{2}}{x}}\right)^2 - \frac{1}{2}\left(\frac{\tan 2x}{\sin\frac{3x}{2}}\right)^2\right] = \frac{5}{2}\left(\frac{4}{9}\right) - \frac{1}{2}\left(\frac{16}{9}\right) = \frac{2}{9}.$$

(2 pt)