KFUPM Summer Term (071) Name: ______ Serial #: ______

• MATH 101 Quiz # 2 ID: #: KEY Sec. #: ______

1. Evaluate the limit if it exists. **If the limit does not exist, explain why**

(a) (5-points) $\lim_{t\to 0}\left[t^{-1} - 5t^{-1}(25+t)^{-1/2}\right] = \lim_{t\to 0}\left[\frac{1}{t} - \frac{5}{t\sqrt{25+t}}\right]$

$$= \lim_{t\to 0} \frac{\sqrt{25+t}-5}{t\sqrt{25+t}} = \lim_{t\to 0} \frac{(\sqrt{25+t}-5)(\sqrt{25+t}+5)}{t\sqrt{25+t}\,(\sqrt{25+t}+5)}$$

$$= \lim_{t\to 0} \frac{25+t-25}{t\sqrt{25+t}\,(\sqrt{25+t}+5)}$$

$$= \lim_{t\to 0} \frac{1}{\sqrt{25+t}\,(\sqrt{25+t}+5)} = \frac{1}{5\,(5+5)}$$

$$= \frac{1}{50}.$$

(b) (5-points) $\lim_{x\to -2/3} \frac{3x+2}{|6x+4|}$.

$$|6x+4| = 2\,|3x+2| = \begin{cases} -2(3x+2), & x \le -\frac{2}{3} \\ 2(3x+2), & x > -\frac{2}{3} \end{cases}$$

$$\Rightarrow \lim_{x\to -\frac{2}{3}^-} \frac{3x+2}{|6x+4|} = \lim_{x\to -\frac{2}{3}^-} \frac{\cancel{3x+2}}{-2\cancel{(3x+2)}} = \lim_{x\to -\frac{2}{3}} -\frac{1}{2} = -\frac{1}{2}$$

while $$\lim_{x\to -\frac{2}{3}^+} \frac{3x+2}{|6x+4|} = \lim_{x\to -\frac{2}{3}^+} \frac{3x+2}{2(3x+2)} = \lim_{x\to -\frac{2}{3}^+} \frac{1}{2} = \frac{1}{2}$$

$\Longrightarrow$ The given limit DNE.

2. (5-points) Find the numbers at which the following function is discontinuous, and classify the type of the discontinuity

$$f(x) = \begin{cases} x+4, & x \leq 2 \\ \dfrac{2x-10}{x-4}, & 2 < x < 4 \\ \dfrac{9}{x}, & x \geq 4 \end{cases}$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x+4) = 6.$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{2x-10}{x-4} = \frac{-6}{-2} = 3$$

Jump discontinuity at $x = 2$

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \frac{2x-10}{x-4} = -\infty$$

$\Longrightarrow$ f has an infinite discontinuity at 4.

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1. Evaluate the limit if it exists. **If the limit does not exist, explain why**

(a) (5-points) $\lim_{t\to 0}\left[3t^{-1}(9+t)^{-1/2} - t^{-1}\right] = \lim_{t\to 0}\left[\frac{3}{t(9+t)^{1/2}} - \frac{1}{t}\right]$

$$= \lim_{t\to 0} \frac{3-\sqrt{9+t}}{t\sqrt{9+t}}$$

$$= \lim_{t\to 0} \frac{(3-\sqrt{9+t})(3+\sqrt{9+t})}{t\sqrt{9+t}\,(3+\sqrt{9+t})}$$

$$= \lim_{t\to 0} \frac{9-(9+t)}{t\sqrt{9+t}\,(3+\sqrt{9+t})}$$

$$= \lim_{t\to 0} \frac{-1}{\sqrt{9+t}\,(3+\sqrt{9+t})} = \frac{-1}{3(3+3)}$$

$$= \frac{-1}{18}.$$

(b) (5-points) $\lim_{x\to -3/4} \frac{4x+3}{|12x+9|}$.

$$|12+9| = 3|4x+3| = \begin{cases} -3(4x+3), & x \leq -3/4 \\ 3(4x+3), & x > -3/4 \end{cases}$$

$$\Rightarrow \lim_{x\to -\frac{3}{4}^-} \frac{4x+3}{|12x+9|} = \lim_{x\to -\frac{3}{4}^-} \frac{(4x+3)}{-3(4x+3)} = \lim_{x\to -\frac{3}{4}^-} -\frac{1}{3} = -1/3$$

While $\lim_{x\to -\frac{3}{4}^+} \frac{4x+3}{|12x+9|} = \lim_{x\to -\frac{3}{4}^+} \frac{4x+3}{3(4x+3)} = \lim_{x\to \frac{3}{4}^+} \frac{1}{3} = 1/3$

$\Rightarrow$ The given limit DNE.

2. (5-points) Find the numbers at which the following function is discontinuous, and classify the type of the discontinuity

$$f(x) = \begin{cases} \frac{5}{x}, & x \leq 3 \\ \frac{3x+4}{x-3}, & 3 < x < 4 \\ 2x-3, & x \geq 4 \end{cases}$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \frac{3x+4}{x-3} = +\infty \Rightarrow f \text{ has}$$

an infinite discontinuity at $x = 3$.

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \frac{3x+4}{x-3} = \frac{16}{1} = 16$$

$$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} (2x-3) = 5$$

$\Rightarrow$ f has a jump discontinuity at $x = 4$.