

2-7 #6, 8, 15 (KEY)

(1)

#6/p.156 To find the slope of the tangent line to the curve  $y = x^3$  at the point  $(-1, 1)$

(i) using definition 1 p.150:  $m = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$  :

$$m = \lim_{x \rightarrow -1} \frac{f(x) - f(-1)}{x - (-1)} = \lim_{x \rightarrow -1} \frac{x^3 - (-1)}{x + 1}$$

$$= \lim_{x \rightarrow -1} \frac{x^3 + 1}{x + 1} = \lim_{x \rightarrow -1} \frac{(x+1)(x^2 - x + 1)}{(x+1)}$$

$$= \lim_{x \rightarrow -1} (x^2 - x + 1) = 3.$$

(ii) using equation 2 p.151  $m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$  :

$$m = \lim_{h \rightarrow 0} \frac{f(-1+h) - f(-1)}{h} = \lim_{h \rightarrow 0} \frac{(-1+h)^3 - (-1)^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-1 + 3h - 3h^2 + h^3 + 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(3 - 3h + h^2)}{h} = \lim_{h \rightarrow 0} (3 - 3h + h^2) = 3.$$

(iii)  $\Rightarrow$  The eqn of the tangent line at  $(-1, 1)$  :

$$y - 1 = 3(x - (-1)) \Rightarrow \boxed{y = 3x + 4}$$

#8/p.156 To find the eqn of the tangent line of the graph of  $y = f(x) = \sqrt{2x+1}$  at  $(4, 3)$  :

$$\text{The slope } m = \lim_{h \rightarrow 0} \frac{f(4+h) - f(4)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{9+h} - 3}{h}$$

2.7

#8/p.156 (cont.)  $m = \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{(\sqrt{9+h}-3)(\sqrt{9+h}+3)}{\sqrt{9+h}+3} \right]$  (2)

$$m = \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{9+h-9}{\sqrt{9+h}+3} \right) = \lim_{h \rightarrow 0} \frac{1}{\sqrt{9+h}+3} = \frac{1}{6}$$

$\Rightarrow$  The required eqn:  $y-3 = \frac{1}{6}(x-4) \Rightarrow$   
 $6y-18 = x-4 \Rightarrow x-6y+14 = 0.$

#15/p.156 Look carefully to the given graph related to the problem on page 156  $\Rightarrow$

- The initial velocity = the slope at  $(0,0) = 0$ .
- The slope at B < the slope at C  $\Rightarrow$  faster at C.
- The slope at A > the slope at B  $\Rightarrow$  speeding up at A and slowing down at B. While before C is speeding up and after C slowing down.
- Between D and E the slope = 0  $\Rightarrow$  no movement.