King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

**CODE 001** **Calculus I** **CODE 001**

# FINAL EXAM

**Semester II, Term 072**

**Date: Saturday, June 07, 2008**

**Net Time Allowed: 180 minutes**

Name: ______________________________

ID: ____________________ Sec: ____________________.

**Check that this exam has 28 questions.**

## Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The value of $\tanh(\ln 3)$ is equal to

(a) $1$

(b) $\frac{4}{5}$

(c) $-\frac{5}{4}$

(d) $-\frac{4}{5}$

(e) $\frac{1}{2}$

2. Let $f(x) = 7 - 3x$ and $\epsilon = 0.03$. A possible value of $\delta$ such that

$$|f(x) + 5| < \epsilon \quad \text{whenever} \quad |x - 4| < \delta$$

is

(a) $0.04$

(b) $0.01$

(c) $0.03$

(d) $-0.01$

(e) $0.1$

3. If $f(x) = \dfrac{\sqrt{4-x^2}}{x}$, then $f'(x) =$

(a) $\dfrac{-4}{x^2\sqrt{4-x^2}}$

(b) $\dfrac{4}{x(4-x^2)^{3/2}}$

(c) $\dfrac{-x^2-\sqrt{4-x^2}}{x^2\sqrt{4-x^2}}$

(d) $\dfrac{x}{\sqrt{4-x^2}}$

(e) $\dfrac{x^2-x-4}{\sqrt{4-x^2}}$

4. The value of the limit $\displaystyle\lim_{x\to-1^-}\frac{|x+1|}{x^2-1}$ is

(a) $\dfrac{1}{2}$

(b) $0$

(c) $+\infty$

(d) $2$

(e) $-\dfrac{1}{2}$

5. The value of the limit $\lim_{x \to -\infty} \tan^{-1}(x^4 - x^2)$ is

(a) $-\infty$

(b) $-\frac{\pi}{2}$

(c) $0$

(d) $1$

(e) $\frac{\pi}{2}$

6. The function $f(x) = \ln(1 - x^2)$ is continuous on

(a) $[-1, 1]$

(b) $(-1, 1)$

(c) $[1, +\infty)$

(d) $(0, +\infty)$

(e) $(-\infty, 0)$

7. Which one of the following statements is **TRUE** about the function $f(x) = \dfrac{x^3}{x^2+1}$?

(a) The line $y = x - 1$ is a slant (oblique) asymptote of $f$

(b) The line $y = 0$ is a horizontal asymptote for $f$

(c) $f$ has no asymptotes

(d) The line $y = x$ is a slant (oblique) asymptote of $f$

(e) $f$ has two vertical asymptotes

8. The linear approximation of $f(x) = e^{-x^2}$ at 0 is

(a) $e^{-x^2} \approx 1 - x$

(b) $e^{-x^2} \approx e^{-1} - 2x$

(c) $e^{-x^2} \approx 0$

(d) $e^{-x^2} \approx 1$

(e) $e^{-x^2} \approx 1 + x$

9. An equation of the tangent line to the curve $y = \sin(\sin x)$ when $x = \pi$ is

(a) $y = x - \pi$

(b) $y = \pi$

(c) $y = \pi(\pi - x)$

(d) $y + x = \pi$

(e) $y = 0$

10. The value(s) of $k$ that will make the function

$$f(x) = \begin{cases} \dfrac{\sin kx}{2x} & \text{if} \quad x > 0 \\ x^2 - k^2 & \text{if} \quad x \leq 0 \end{cases}$$

continuous on $(-\infty, +\infty)$ is (are)

(a) $-1$ and $\frac{1}{2}$

(b) $-\frac{1}{2}$

(c) $\frac{1}{2}$

(d) $0$ and $-\frac{1}{2}$

(e) $2$

11. If $y = \tanh^{-1}(\cosh(2x))$, then $y' =$

(a) $\tanh(2x)$

(b) $-2\ \text{sech}(2x)$

(c) $-2\ \text{csch}(2x)$

(d) $2x\ \text{csch}(2x)$

(e) $2\ \text{sech}(2x)$

12. If $y^x = (2-x)^y$, then $y'$ at $(1,1)$ is equal to

(a) $-1$

(b) $0$

(c) $1$

(d) $-\ln 2$

(e) $\ln 2$

13. Using differentials (or, equivalently, a linear approximation), the value of $\sqrt{80.9}$ is approximately equal to

(a) $9 - \frac{1}{90}$

(b) $9 - \frac{1}{180}$

(c) $9 - \frac{1}{360}$

(d) $9 - \frac{1}{20}$

(e) $9 - \frac{1}{10}$

14. The critical number(s) of the function $f(x) = x^{1/3} - x^{-2/3}$ is (are)

(a) $x = 0$ only

(b) $x = -2$ only

(c) $x = 0$ and $x = -1$

(d) $x = -2$ and $x = 0$

(e) $x = -1$ only

15. The absolute maximum and absolute minimum values of the function

$$f(x) = \sin x - \cos x$$

on the interval $[0, \pi]$ are respectively

(a) 0 and $-1$

(b) $\sqrt{2}$ and $-1$

(c) 1 and 0

(d) $\dfrac{\sqrt{2}}{2}$ and $-1$

(e) 1 and $-1$

16. If $f''(x) = 12x$, $f(0) = 6$, $f'(0) = 0$, then the sum of the coefficients of $f$ is

(a) 8

(b) 2

(c) 9

(d) 6

(e) 12

17. Newton's Method is used to find a root of the equation

$$\sin x - \tan(2x) = 0.$$

If the first approximation is $x_1 = \frac{\pi}{2}$, then the second approximation $x_2$ is equal to

(a) $\frac{\pi}{2} + 1$

(b) $\frac{\pi}{2}$

(c) $0$

(d) $\frac{\pi - 1}{2}$

(e) $\frac{\pi + 1}{2}$

18. A stone dropped in a still pond generates a circular wave whose radius increases at a constant rate of 3 ft/s. The rate at which the area of the circular wave is increasing after 10 s is (ft: feet; s: seconds)

(a) $60\pi$ ft$^2$/ s

(b) $90\pi$ ft$^2$/ s

(c) $180\pi$ ft$^2$/ s

(d) $30\pi$ ft$^2$/ s

(e) $270\pi$ ft$^2$/ s

19. $\lim\limits_{x\to 0} \dfrac{x - \sin(x^2)}{x^2 - x} =$

(a) $-1$

(b) $0$

(c) $1/2$

(d) $1$

(e) $+\infty$

20. If $y\sin(x^2) = x\sin(y^2)$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{\sin(y^2) + xy\cos(x^2)}{\sin(y^2) - xy\cos(x^2)}$

(b) $\dfrac{\sin(y^2)}{\sin(x^2) - 2xy\cos(y^2)}$

(c) $\dfrac{\cos(y^2) - 2xy\sin(x^2)}{\cos(x^2) - 2xy\sin(y^2)}$

(d) $\dfrac{\sin(x^2) + \cos(y^2)}{2xy + \sin(y^2)}$

(e) $\dfrac{\sin(y^2) - 2xy\cos(x^2)}{\sin(x^2) - 2xy\cos(y^2)}$

21. $\lim_{x \to 1} (2-x)^{\tan\left(\frac{\pi}{2}x\right)} =$

(a) $0$

(b) $e^{\pi}$

(c) $1$

(d) $e^{2/\pi}$

(e) $e^{-2/\pi}$

22. The sum of the coordinates of the point $P$ on the curve $y = x^2$ that is **closest** to the point $\left(2, \frac{1}{2}\right)$ is equal to

(a) $2$

(b) $0$

(c) $1$

(d) $5$

(e) $\frac{5}{2}$

23. The most general antiderivative of $f(x) = \sqrt[4]{x^3} - \sin x + \dfrac{3}{x}$ is

(a) $\dfrac{4}{7}x^{7/4} + \cos x + 3\ln x + C$

(b) $\dfrac{6}{7}x^{7/6} + \cos x + 3\ln|x| + C$

(c) $\dfrac{4}{7}x^{7/4} - \cos x + 3\ln|x| + C$

(d) $\dfrac{4}{7}x^{7/4} + \cos x + 3\ln|x| + C$

(e) $\dfrac{5}{7}x^{7/5} + \cos x + 3\ln|x|$

24. The function $f(x) = 3x^5 - 5x^3 + 3$

(a) has a local maximum at $x = 1$

(b) is increasing on $(0, +\infty)$

(c) has a local minimum at $x = 0$

(d) is decreasing on $(-\infty, 1)$

(e) is decreasing on $(-1, 1)$

25. Which one of the following statements is **TRUE**?

(a) If $f'(x) = g'(x)$ for all $x$ in an interval $(a, b)$, then $f(x) = g(x)$ for all $x$ in $(a, b)$

(b) If $f$ is differentiable and $f(-1) = f(1)$ then there is a number $c$ such that $|c| < 1$ and $f'(c) = 0$

(c) If $f(a) \geq f(x)$ when $x$ is near $a$, then $f$ has a local minimum at $a$

(d) The function $f(x) = x^3 + 4x + 4$ has no real roots in the interval $[-1, 1]$

(e) If $f$ has a local maximum or a local minimum at $c$, then $f'(c) = 0$

26. The graph of $f(x) = \dfrac{x^2}{x^2 - 1}$

(a) has two inflection points

(b) is concave down on the interval $(0, +\infty)$

(c) is concave up on the interval $(-\infty, 1)$

(d) is concave up on the intervals $(-\infty, -1)$ and $(1, +\infty)$

(e) has one inflection point

27. If $f(1) = -2$ and $f'(x) \leq 7$ for all values of $x$, then the largest possible value that $f(3)$ can have is (Hint: Use the Mean Value Theorem)

(a) 10

(b) 9

(c) 12

(d) 14

(e) 11

28. Which one of the following statements is **FALSE** about the function $f(x) = \dfrac{\ln x}{x}$?

(a) $f$ is concave up on $(10, +\infty)$

(b) $f$ is decreasing on $(e, +\infty)$

(c) The absolute maximum value of $f$ is $\dfrac{1}{e}$

(d) The graph of $f$ has inflection point at $x = e$

(e) $f$ has one inflection point

| Q | MM | V1 | V2 | V3 | V4 |
|---|---|---|---|---|---|
| 1 | a | b | e | a | a |
| 2 | a | b | c | e | d |
| 3 | a | a | c | d | d |
| 4 | a | a | d | c | e |
| 5 | a | e | a | b | c |
| 6 | a | b | c | d | a |
| 7 | a | d | d | c | e |
| 8 | a | d | c | a | c |
| 9 | a | d | d | a | c |
| 10 | a | d | a | a | e |
| 11 | a | c | a | a | a |
| 12 | a | a | b | e | c |
| 13 | a | b | c | a | e |
| 14 | a | b | a | d | d |
| 15 | a | b | c | c | c |
| 16 | a | a | a | c | c |
| 17 | a | e | d | a | b |
| 18 | a | c | c | d | e |
| 19 | a | a | b | b | a |
| 20 | a | e | e | a | c |
| 21 | a | d | b | b | a |
| 22 | a | a | b | d | d |
| 23 | a | d | c | d | b |
| 24 | a | e | c | c | e |
| 25 | a | b | d | d | b |
| 26 | a | d | b | d | d |
| 27 | a | c | b | d | c |
| 28 | a | d | e | b | d |