KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES

MATH 101 – Final Exam

Sunday – January 20, 2008

Test Code: Master **Duration: 3 Hours**

Student's Name: ……………………………………………………………

ID #: ………………………….. **Section #: ……………………**

Important Instructions:

1. All types of CALCULATORS, PAGERS, OR MOBILES ARE NOT ALLOWED to be with you during the examination.

2. Use an HB 2 pencil.

3. Use a good eraser. Do not use the eraser attached to the pencil.

4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.

5. When you bubble your ID number and Section number, be sure that bubbles match with the number that you write.

6. The test Code Number is already typed and bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.

7. When bubbling, make sure that the bubbled space is fully covered.

8. When erasing a bubble, make sure that you do not leave any trace of penciling.

9. Check that the exam paper has 28 questions.

1. $\lim_{x \to \frac{\pi}{3}} \dfrac{\tan x - \sqrt{3}}{x - \dfrac{\pi}{3}} =$

(a) -1

(b) ∞

(c) 3

(d) 4

(e) 1

2. The equation of the horizontal asymptote of the function $f(x) = \dfrac{\sinh x}{e^x}$ is

(a) $y = 1$

(b) $y = \dfrac{1}{2}$

(c) $y = 0$

(d) $y = -\dfrac{1}{2}$

(e) $x = -\dfrac{3}{2}$

3. The sum of the slopes of the lines through the point $(1, -3)$ that are tangent to the parabola $y = x^2$ is

(a) 4

(b) -6

(c) 0

(d) 6

(e) -2

4. If $f(t) = t^2 + 3t + 5$ is the position of an object at time t, where $f(t)$ is in feet and t in seconds, then the average velocity of the object over the interval $[1, 3]$ is

(a) 6 ft/sec

(b) 7 ft/sec

(c) 5 ft/sec

(d) $-2/3$ ft/sec

(e) $23/2$ ft/sec

5. If $g(x) = \dfrac{x^2}{f(\sqrt{x})}$, $f(2) = 1$, and $f'(2) = -1$, then $g'(4) =$

(a) 12

(b) -4

(c) 8

(d) 0

(e) 6

6. $\displaystyle\lim_{x \to 0} \frac{\tan^2 2x}{x \sin x} =$

(a) 4

(b) ∞

(c) does not exist

(d) 1

(e) 2

7. If the function $f(x) = \begin{cases} kx^2 & \text{if } x \leq 2 \\ 2x + k^2 & \text{if } x > 2 \end{cases}$
is continuous everywhere then k equals

(a) -2 only

(b) $\dfrac{1}{2}$ only

(c) 2 and -2

(d) 2 only

(e) -3 only

8. If $x^2y + 3y + \sin(xy) = 6$, then $y'(0) =$

(a) 0

(b) $-\dfrac{3}{2}$

(c) -3

(d) $-\dfrac{2}{3}$

(e) -2

9. Which of the following statements is **TRUE** about the function $f(x) = x^{2/3} + 5$?

(a) $f(x)$ is discontinuous at $x = 0$

(b) $f(x)$ is differentiable at $x = 0$

(c) $f(x)$ has no criticial number

(d) $f(x)$ has a vertical asymptote

(e) $f(x)$ has a vertical tangent at $x = 0$

10. If $f(x) = \begin{cases} |2x-5| & \text{if } x < 1 \\ -1 & \text{if } x = 1 \\ \sqrt{8x+1} & \text{if } x > 1 \end{cases}$, then $\lim_{x \to 1} f(x)$

(a) is equal to 3

(b) is equal to -1

(c) is equal to -3

(d) does not exist

(e) is equal to 7

11. Using differentials, $(8.06)^{2/3}$ can be approximated to:

(a) 4.01

(b) 4.02

(c) 4.03

(d) 3.98

(e) 4.08

12. The function $f(x) = x - \sqrt{x}, \quad 0 \leq x \leq 4$ has

(a) absolute maximum 2 and no absolute minimum

(b) absolute maximum 0 and absolute minimum $-\frac{1}{4}$

(c) absolute maximum 2 and absolute minimum $-\frac{1}{4}$

(d) absolute maximum 2 and absolute minimum 0

(e) absolute maximum 4 and absolute minimum $\frac{1}{4}$

13. The linear approximation to $\dfrac{1}{1+x}$ at $x = 0$ is

(a) $2x - \dfrac{1}{2}$

(b) $1 - x$

(c) $x - 1$

(d) $2 - \dfrac{1}{2}x$

(e) x

14. If $\cosh(\ln(2x)) = 1$, then

(a) $\cosh(2x) = e$

(b) $\cosh(2x) = \dfrac{1}{2\sqrt{e}}$

(c) $\cosh(2x) = \dfrac{1}{e^2}$

(d) $\cosh(2x) = \dfrac{e^2+1}{2e}$

(e) $\cosh(2x) = \dfrac{e+1}{2}$

15. If $f(x) = x^{50} + \ln(x+1)$, then $f^{(101)}(0) =$

(a) $-(100)!$

(b) $-(101)!$

(c) $(101)!$

(d) $(100)!$

(e) 0

16. Given the function $f(x) = \dfrac{1}{x^2+1}$ with $f'(x) = \dfrac{-2x}{(x^2+1)^2}$ and $f''(x) = \dfrac{6x^2-2}{(x^2+1)^3}$ which of the following statements is **TRUE** about the graph of $f(x)$?

(a) has one inflection point only

(b) concave up on $(-\infty, 0)$ and concave down on $(0, \infty)$

(c) concave down on $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$ & $\left(\frac{1}{\sqrt{3}}, \infty\right)$, and up on $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(d) concave up on $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$ & $\left(\frac{1}{\sqrt{3}}, \infty\right)$, and down on $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(e) concave up on $\left(-\infty, -\sqrt{3}\right)$ & $\left(\sqrt{3}, \infty\right)$, and down on $(-\sqrt{3}, \sqrt{3})$

17. If $f(x) = \log_3 \sqrt{\dfrac{x-2}{x^2+1}}$, then $f'(3) =$

(a) $\dfrac{1}{5\ln 3}$

(b) $-\dfrac{3}{5\ln 3}$

(c) $\dfrac{3}{5\ln 3}$

(d) $\dfrac{1}{4\ln 3}$

(e) $\dfrac{1}{5}$

18. For the graph of the curve $y = 3x^5 - 5x^3 + 3$, which one of the following is **FALSE**?

(a) The graph has the local minimum at $(0, 3)$.

(b) The graph is increasing over the interval $(1, \infty)$.

(c) The graph has the local minimum at $(1, 1)$.

(d) The graph has the local maximum at $(-1, 5)$.

(e) The graph is decreasing over the interval $(-1, 1)$.

19. If c is a number that satisfies the conclusion of the mean value theorem on the interval $[0, 1]$ for the function $f(x) = x^3 + 2x + 1$, then $12c^2 + 1$ is equal to

(a) 3

(b) 5

(c) 7

(d) Undefined (The mean value theorem does not apply)

(e) 2

20. $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right) =$

(a) $+\infty$

(b) $-\frac{1}{2}$

(c) $\frac{1}{2}$

(d) 0

(e) 1

21. If $y = (1+\cos x)^{\frac{1}{x+1}}$ then $y'(0) =$

(a) $-\ln 2$

(b) 1

(c) $\ln 4$

(d) $-\ln 8$

(e) $-\ln 4$

22. If $y = \tan^{-1}(\operatorname{csch}\sqrt{x})$, then $y' =$

(a) $-\dfrac{\operatorname{sech}\sqrt{x}}{2\sqrt{x}(\operatorname{csch}\sqrt{x})}$

(b) $-\dfrac{\operatorname{csch}\sqrt{x}}{2\sqrt{x}}$

(c) $-\dfrac{\operatorname{sech}\sqrt{x}}{2\sqrt{x}}$

(d) $\dfrac{\operatorname{sech}\sqrt{x}}{2\sqrt{x}}$

(e) $\dfrac{\operatorname{csch}\sqrt{x}}{2\sqrt{x}}$

23. Starting with $x_1 = 1$, the approximation x_3 to the root of the equation $x + \ln x = 0$ obtained by using Newton's method is

(a) $\dfrac{1 + \ln\left(\frac{1}{2}\right)}{3}$

(b) $\dfrac{1 + \ln 2}{3}$

(c) $\dfrac{1}{2}$

(d) $\dfrac{3}{5} + \dfrac{3}{5}\ln\dfrac{2}{3}$

(e) $\ln 2$

24. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of 3 cm^3/s, then when the water is 5 cm deep, the water level is rising at a rate of

(Volume of cone $= \frac{1}{3}\pi r^2 h$)

(a) $\dfrac{9\pi}{8}$ cm/s

(b) $\dfrac{4}{3\pi}$ cm/s

(c) $\dfrac{8}{9\pi}$ cm/s

(d) $\dfrac{3}{4\pi}$ cm/s

(e) $\dfrac{8\pi}{9}$ cm/s

25. Let $f(x)$ be the function defined by $f(x) = 2x - 1$ and $\epsilon = 0.002$. The largest possible δ such that $|f(x) - 3| < \epsilon$ whenever $|x - 2| < \delta$ is

(a) 0.002

(b) 0.003

(c) 0.005

(d) 0.001

(e) 0.01

26. If a particle moves in a straight line and has acceleration given by $a(t) = 6t+4$, its initial velocity is $v(0) = -6$ cm/s, and its initial displacement is $s(0) = 0$ cm, then $s(1)$ equals

(a) -3 cm

(b) 6 cm

(c) 1 cm

(d) -6 cm

(e) 12 cm

27. If $y'' = \sin x + x^2 - x$, $y'(0) = 1$ and $y(0) = 3$, then $y =$

(a) $\sin x - \frac{1}{6}x^3 + \frac{1}{12}x^4 + 3$

(b) $\sin x - \frac{1}{6}x^3 + \frac{1}{12}x^4 + 2x + 3$

(c) $-\sin x - \frac{1}{6}x^3 + \frac{1}{12}x^4 + 2x + 3$

(d) $\cos x - \frac{1}{6}x^3 + \frac{1}{12}x^4$

(e) $\cos x - \frac{1}{6}x^3 + \frac{1}{12}x^4 + 2x + 3$

28. If 2700 cm^2 of material is used to make a rectangular box with a square bottom and no top, then the largest possible volume for the box is

(a) 30 cm^3

(b) 16900 cm^3

(c) 13500 cm^3

(d) 500 cm^3

(e) 1540 cm^3