KFUPM SEM II (Term 052) Name: ______________ Serial #: KEY

• MATH 102-4-8 Quiz # 2 ID: #: ______________ Section # ——

SOLVE THREE PROBLEMS ONLY

1. (5-points) Evaluate $\lim\limits_{\max \Delta x_k \to 0} \sum\limits_{k=1}^{n} \sqrt{4x_k^* - x_k^{*2}}\, \Delta x_k$ over the interval $[0, 4]$ by expressing it as a definite integral and applying appropriate formula from geometry.

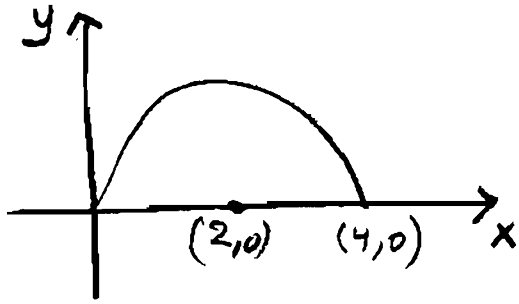
The given Limit $= \int_0^4 \sqrt{4x - x^2}\, dx = \int_0^4 \sqrt{4 - (x-2)^2}\, dx$

The graph of $y = \sqrt{4 - (x-2)^2}\, dx$ is a semicircle of a circle with center at $(2, 0)$ and radius 2

The given Limit = The area of the semicircle

$= \frac{1}{2}\left(\pi (2)^2\right)$

$= 2\pi$



2. (5-points) Find the average value of the function $f(x) = \dfrac{\cos x \; e^{\sqrt{\sin x}}}{\sqrt{\sin x}}$ over the interval $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$.

average of $f = \dfrac{1}{\frac{\pi}{2} - \frac{\pi}{6}} \displaystyle\int_{\pi/6}^{\pi/2} \dfrac{\cos x \; e^{\sqrt{\sin x}}}{\sqrt{\sin x}}\, dx$

Let $u = \sqrt{\sin x} \Rightarrow du = \dfrac{\cos x}{2\sqrt{\sin x}}\, dx$

when $x = \frac{\pi}{6} \Rightarrow u = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$ and when

$u = \frac{\pi}{2} \Rightarrow u = 1 \Longrightarrow$

The average $= \dfrac{3}{\pi} \displaystyle\int_{1/\sqrt{2}}^{1} e^{u} (2\,du)\, du$

$= \dfrac{6}{\pi}\left[e - e^{1/\sqrt{2}}\right].$

• 3. (5-points) Evaluate $\displaystyle\int_1^{\sqrt[4]{e}} \frac{dx}{x\sqrt{1-4(\ln x)^2}} = \int_1^{\sqrt[4]{e}} \frac{dx}{x\sqrt{1-(2\ln x)^2}}$

Put $u = 2\ln x \Rightarrow du = \frac{2}{x}\, dx$

when $x = 1 \Rightarrow u = 0$ and when $u = \sqrt[4]{e}$

$\Rightarrow u = \frac{2}{4}\ln e = \frac{1}{2} \Rightarrow$

The given integral $= \displaystyle\int_0^{1/2} \frac{\frac{1}{2}\, du}{\sqrt{1-u^2}}$

$= \frac{1}{2}\sin^{-1} u \Big]_0^{1/2} = \frac{1}{2}\left[\sin^{-1}\frac{1}{2} - \sin^{-1} 0\right]$

$= \frac{1}{2}\left[\frac{\pi}{6} - 0\right] = \frac{\pi}{12}$.

4. (5-points) If $F(x) = \displaystyle\int_{\sqrt{x}}^{\sqrt{2x}} \sin \pi t^2\, dt$, find the value of $F'\left(\frac{1}{4}\right)$.

$F'(x) = (\sin \pi(2x))\,\frac{2}{2\sqrt{2x}} - (\sin \pi x)\,\frac{1}{2\sqrt{x}}$

$= \dfrac{\sin 2\pi x}{\sqrt{2x}} - \dfrac{\sin \pi x}{2\sqrt{x}}$

$\Rightarrow F'\left(\frac{1}{4}\right) = \dfrac{\sin\frac{\pi}{2}}{\sqrt{\frac{1}{2}}} - \dfrac{\sin\frac{\pi}{x}}{2\sqrt{\frac{1}{4}}}$

$= \sqrt{2} - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$.

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MATH 102-4 Quiz # 2 ID: #: ______________ Section# ____

SOLVE THREE PROBLEMS ONLY

1. (5-points) Evaluate $\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} \sqrt{6x_k^* - x_k^{*2}}\, \Delta x_k$ over the interval $[0, 6]$ by expressing it as a definite integral and applying appropriate formula from geometry.

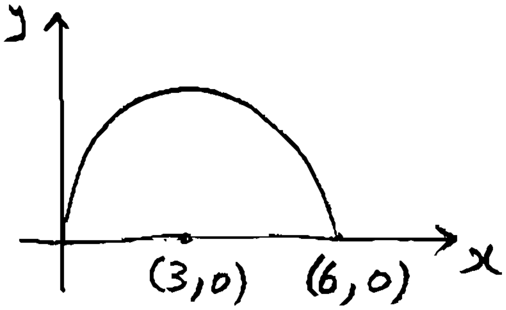
The given Limit $= \int_0^6 \sqrt{6x - x^2}\, dx = \int_0^6 \sqrt{9 - (x-3)^2}\, dx$

The graph of $y = \sqrt{9 - (x-3)^2}$ is a semicircle of a circle with center at $(3\ 0)$ and radius 3

The given limit = The area of the semicircle

$= \frac{1}{2}\left(\pi (3)^2\right)$

$= \frac{9}{2}\pi$



2. (5-points) Find the average value of the function $f(x) = \dfrac{\sin x\ e^{\sqrt{\cos x}}}{\sqrt{\cos x}}$ over the interval $\left[0, \frac{\pi}{3}\right]$.

average of $f = \dfrac{1}{\frac{\pi}{3} - 0} \displaystyle\int_0^{\pi/3} \dfrac{\sin x\ e^{\sqrt{\cos x}}}{\sqrt{\cos x}}\, dx$

Let $u = \sqrt{\cos x} \Rightarrow du = \dfrac{-\sin x}{2\sqrt{\cos x}}\, dx$

when $x = 0 \Rightarrow u = 1$, and when $x = \frac{\pi}{3} \Rightarrow u = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow$ The average $= \dfrac{3}{\pi} \displaystyle\int_1^{\frac{1}{\sqrt{2}}} e^{u}\,(-2\,du)$

$= -\dfrac{6}{\pi}\left[e^{u}\right]_1^{\frac{1}{\sqrt{2}}} = -\dfrac{6}{\pi}\left[e^{\frac{1}{\sqrt{2}}} - e\right]$

$= \dfrac{6}{\pi}\left[e - e^{1/\sqrt{2}}\right].$

3. (5-points) Evaluate $\int_1^{\sqrt[6]{e}} \frac{dx}{x\sqrt{1-9(\ln x)^2}}$. $= \int_1^{\sqrt[6]{e}} \frac{dx}{x\sqrt{1-(3\ln x)^2}}$

Put $u = 3\ln x \Rightarrow du = \frac{3}{x}\,dx$

when $x=1 \Rightarrow u=0$ and when $u = \sqrt[6]{e}$

$\Rightarrow u = \frac{3}{6}\ln e = \frac{1}{2} \Rightarrow$

The given integral $= \int_0^{1/2} \frac{\frac{1}{3}\,du}{\sqrt{1-u^2}}$

$= \frac{1}{3}\sin^{-1}u\Big]_0^{1/2} = \frac{1}{3}\left[\sin^{-1}\frac{1}{2} - \sin^{-1}0\right]$

$= \frac{1}{3}\left[\frac{\pi}{6} - 0\right] = \frac{\pi}{18}$

4. (5-points) If $F(x) = \int_{\sqrt{x}}^{\sqrt{2x}} \cos\pi t^2\,dt$, find the value of $F'\left(\frac{1}{4}\right)$.

$F'(x) = \left(\cos\pi(2x)\right)\frac{2}{2\sqrt{2x}} - \left(\cos\pi x\right)\frac{1}{2\sqrt{x}}$

$= \frac{\cos 2\pi x}{\sqrt{2x}} - \frac{\cos\pi x}{2\sqrt{x}}$

$\Rightarrow F'\left(\frac{1}{4}\right) = \frac{\cos\frac{\pi}{2}}{\sqrt{\frac{1}{2}}} - \frac{\cos\frac{\pi}{4}}{2\sqrt{\frac{1}{4}}}$

$= 0 - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}.$