KFUPM SEM II (Term 052) Name: ____________ Serial #: KEY

• MATH 102-4 Quiz # 5 ID: #: ____________

1. (5-points each) Evaluate the integral that converge

(a) $\displaystyle\int_{e/3}^{\infty} \frac{1}{x(\ln 3x)^4}\,dx. = \lim_{t\to\infty} \int_{e/3}^{t} \frac{1}{x(\ln 3x)^4}\,dx = I$

Put $u = \ln 3x \Rightarrow du = \frac{1}{x}\,dx$, $\boxed{x = e/3 \Rightarrow u = 1}$ $\boxed{\text{and } x = t \Rightarrow u = \ln 3t} \Longrightarrow$

$$I = \lim_{t\to\infty} \int_{1}^{\ln 3t} \frac{1}{u^4}\,du = \lim_{t\to\infty} \left[-\frac{1}{3u^3}\right]_1^{\ln 3t}$$

$$= \lim_{t\to\infty} \left[-\left(\frac{1}{3(\ln 3t)^3} - \frac{1}{3}\right)\right] = 0 + \frac{1}{3} = \frac{1}{3}$$

$\Rightarrow$ the improper integral converges and has the value $\frac{1}{3}$.

(b) $\displaystyle\int_{0}^{\pi/6} \frac{\cos x}{\sqrt{1-2\sin x}}\,dx. = \lim_{t\to\frac{\pi}{6}^-} \int_{0}^{t} \frac{\cos x}{\sqrt{1-2\sin x}}\,dx = I$

Put $u = \sqrt{1-2\sin x} \Rightarrow du = \frac{-\cancel{2}\cos x}{\cancel{2}\sqrt{1-2\sin x}}\,dx$

$\boxed{x = 0 \Rightarrow u = 1}$, $\boxed{x = t \Rightarrow u = \sqrt{1-2\sin t}} \Longrightarrow$

$$I = \lim_{t\to\frac{\pi}{6}^-} \int_{1}^{\sqrt{1-2\sin t}} -\,du = \lim_{t\to\frac{\pi}{6}^-} -\left[u\right]_1^{\sqrt{1-2\sin t}}$$

$$= \lim_{t\to\frac{\pi}{6}^-} -\left[\sqrt{1-2\sin t} - 1\right] = 1$$ converges and has the value 1

P.T.O. →

2. (5-points) Find the Maclaurin polynomial P_n for $f(x) = \dfrac{1}{1+2x}$. Show your steps. [Hint: First find P_0, P_1, P_2, P_3 and P_4 and then find P_n and write it in a closed form.]

$$f(x) = \frac{1}{1+2x} \Longrightarrow f(0) = 1$$

$$f'(x) = \frac{-2}{(1+2x)^2} \Rightarrow f'(0) = -2$$

$$f''(x) = \frac{2^2\,(2)}{(1+2x)^3} \Rightarrow f''(0) = 2^2\,(2)$$

$$f'''(x) = \frac{-\,2^3\,(2\cdot 3)}{(1+2x)^4} \Rightarrow f'''(0) = -2^3\,(2\cdot 3)$$

$$f^{(4)}(x) = \frac{2^4\,(2\cdot 3\cdot 4)}{(1+2x)^5} \Rightarrow f^{(4)}(0) = 2^4\,(2\cdot 3\cdot 4)$$

and so on $\Longrightarrow$

- $P_0(x) = f(0) \Longrightarrow P_0(x) = 1$
- $P_1(x) = f(0) + f'(0)\,x \Rightarrow P_1(x) = 1 - 2x$
- $P_2(x) = f(0) + f'(0)\,x + \dfrac{f''(0)}{2!}\,x^2 \Longrightarrow$

  $P_2(x) = 1 - 2x + 2^2\,x^2$
- $P_3(x) = 1 + f'(0)x + f''(0)\,x^2 + f'''(0)\,x^3 \Longrightarrow$

  $P_3(x) = 1 - 2x + 2^2x^2 - 2^3\,x^3$ and hence
- $P_4(x) = 1 - 2x + 2^2x^2 - 2^3x^3 + 2^4x^4 \Longrightarrow$
- $P_n = 1 - 2x + 2^2x^2 - 2^3x^3 + \cdots + (-1)^n\,2^n x^n, \quad n = 0,1,2,\ldots$

$$= \sum_{k=0}^{n} (-1)^k\,2^k\,x^k$$

P.T.O. $\longrightarrow$

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(a) $\displaystyle\int_{e/4}^{\infty} \frac{1}{x(\ln 4x)^3}\,dx. = \lim_{t\to\infty} \int_{e/4}^{t} \frac{1}{x(\ln 4x)^3}\,dx = I$

Put $u = \ln 4x \Rightarrow du = \frac{1}{x}\,dx$, $\boxed{x = e/4 \Rightarrow u = 1}$

and $\boxed{x = t \Rightarrow u = \ln 4t}$ $\Rightarrow$

$$I = \lim_{t\to\infty} \int_{1}^{\ln 4t} \frac{1}{u^3}\,du = \lim_{t\to\infty} \left[\frac{-1}{2u^2}\right]_1^{\ln 4t}$$

$$= \lim_{t\to\infty} -\left[\frac{1}{2\ln 4t} - \frac{1}{2}\right] = 0 + \frac{1}{2} = \frac{1}{2}$$

$\Rightarrow$ the improper integral converges and has the value $\frac{1}{2}$.

(b) $\displaystyle\int_{\frac{\pi}{3}}^{\pi/2} \frac{\sin x}{\sqrt{1-2\cos x}}\,dx. = \lim_{t\to\frac{\pi}{3}^+} \int_{t}^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{1-2\cos x}}\,dx = I$

Put $u = \sqrt{1-2\cos x} \Rightarrow du = \frac{\cancel{2}\sin x}{\cancel{2}\sqrt{1-2\cos x}}\,dx \Rightarrow$

$\boxed{x = t \Rightarrow u = \sqrt{1-2\cos t}}$, $\boxed{x = \frac{\pi}{2} \Rightarrow u = 1}$ $\Rightarrow$

$$I = \lim_{t\to\frac{\pi}{3}^+} \int_{\sqrt{1-2\cos t}}^{1} du = \lim_{t\to\frac{\pi}{3}^+} \left[u\right]_{\sqrt{1-2\cos t}}^{1}$$

$$= \lim_{t\to\frac{\pi}{3}^+} \left[1 - \sqrt{1-2\cos t}\right] = 1 \text{ Converges}$$

and has the value 1

P.T.O. $\rightarrow$

2. (5-points) Find the Maclaurin polynomial P_n for $f(x) = \dfrac{1}{1+3x}$. Show your steps. **Show your steps. [Hint: First find P_0, P_1, P_2, P_3 and P_4 and then find P_n and write it in a closed form.]**

$$f(x) = \frac{1}{1+3x} \Rightarrow f(0) = 1$$

$$f'(x) = \frac{-3}{(1+3x)^2} \Rightarrow f'(0) = -3$$

$$f''(x) = \frac{3^2(2)}{(1+3x)^3} \Rightarrow f''(0) = 3^2(2)$$

$$f'''(x) = \frac{-3^3(2.3)}{(1+3x)^4} \Rightarrow f'''(0) = -3^3(2.3)$$

$$f^{(4)}(x) = \frac{3^4(2.3.4)}{(1+3x)^5} \Rightarrow f^{(4)}(0) = 3^4(2.3.4)$$

and so on $\Rightarrow$

- $P_0(x) = f(0) \Rightarrow P_0(x) = 1$
- $P_1(x) = f(0) + f'(0) \Rightarrow$ • $P_1(x) = 1 - 3x$

$P_2(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} \Rightarrow$

- $P_2(x) = 1 - 3x + 3^2x^2$

$P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 \Rightarrow$

- $P_4(x) = 1 - 3x + 3^2x^2 - 3^3x^3 + 3^4x^4 \Rightarrow$
- $P_n(x) = 1 - 3x + 3^2x^2 - 3^3x^3 + \cdots + (-1)^n 3^n x^n, \; n = 0, 1, \ldots$

$$= \sum_{k=0}^{\infty} (-1)^k 3^k x^k$$

P.T.O. $\longrightarrow$