KFUPM **Semester II (052)** **Name** ——— **Serial** ———
Math 102-4, 8 **Quiz # 3** **ID#:** KEY **Sec #:** ———

Solve three problem only

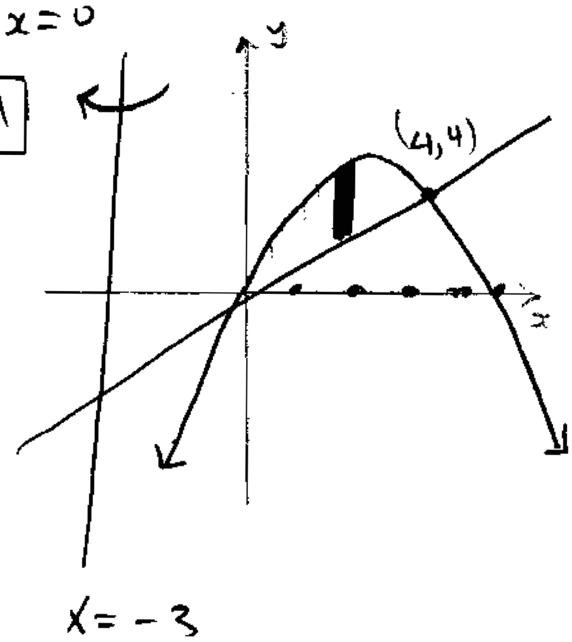
1. (5- Points) Use cylindrical shells to **set up** an integral which represents the volume of the solid generated by revolving the area enclosed by the graphs of $y = 5x - x^2$ and $y = x$ about the vertical line $x = -3$.

Pts of intersection $5x - x^2 = x \Rightarrow x^2 - 4x = 0$

$x(x-4) = 0 \Rightarrow \boxed{x = 0,\ y = 0}, \boxed{x = 4,\ y = 4}$

$$dV = 2\pi \underbrace{(x+3)}_{\text{ave. radius}} \underbrace{(5x - x^2 - x)}_{\text{height}} \underbrace{dx}_{\text{thickness}}$$

$$\Rightarrow V = \int_0^4 2\pi (x+3)(4x - x^2)\, dx$$



2. (5- Points) Find the exact arc length of the parametric curve $x = e^{3t}\cos 2t,\ y = e^{3t}\sin 2t,\ 0 \le t \le \pi/4$.

$$\frac{dx}{dt} = 3e^{3t}\cos 2t - 2e^{3t}\sin 2t \ ; \ \frac{dy}{dt} = 3e^{3t}\sin 2t + 2e^{3t}\cos 2t$$

$$\Rightarrow \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 9e^{6t}(\cos^2 2t + \sin^2 2t) + 12e^{6t}(\cos 2t \sin 2t - \sin 2t \cos 2t)$$
$$+ 4e^{6t}(\sin^2 2t + \cos^2 2t) = 13e^{6t}$$

$$\Rightarrow dL = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{13e^{6t}} = \sqrt{13}\, e^{3t} \Rightarrow$$

$$L = \sqrt{13}\int_0^{\pi/4} e^{3t}\, dt = \frac{\sqrt{13}}{3}\left[e^{3t}\right]_0^{\pi/4} \Rightarrow$$

$$L = \frac{\sqrt{13}}{3}\left(e^{\frac{3\pi}{4}} - 1\right).$$

P.T.O. $\longrightarrow$

3. (5- Points) find the area of the surface generated by revolving the curve $y=\sqrt{16-x^2}$, $-2 \le x \le 2$ about the $x-$axis.

$$\frac{dy}{dt} = \frac{-2x}{2\sqrt{16-x^2}} \Rightarrow 1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{x^2}{16-x^2} = \frac{16}{16-x^2}$$

$$\Rightarrow dS = 2\pi y \, dL = 2\pi\sqrt{16-x^2}\sqrt{\frac{16}{16-x^2}}\,dx \Rightarrow$$

$$dS = 8\pi\,dx \Rightarrow S = \int_{-2}^{2} 8\pi\,dx \Rightarrow$$

$$S' = 8\pi\left[x\right]_{-2}^{2} = 32\pi.$$

4. (5-Points) (a) Evaluate $\lim_{x\to-\infty} \coth\frac{x}{3}$. $= \lim_{x\to-\infty} \frac{e^{\frac{x}{3}}+e^{-\frac{x}{3}}}{e^{\frac{x}{3}}-e^{-\frac{x}{3}}}$

$$= \lim_{x\to-\infty} \frac{e^{\frac{2x}{3}}+1}{e^{\frac{2x}{3}}-1} = \frac{0+1}{0-1} = -1$$

(b) Evaluate $\int_0^{\ln 9} \cos h\frac{x}{2}\,dx$. [write your answer in simplest form]

Put $u = \frac{x}{2} \Rightarrow du = \frac{1}{2}dx$, $x=0 \Rightarrow u=0$, $x=\ln 9 \Rightarrow u = \ln 3$

$$\Rightarrow \int_0^{\ln 9} \cosh\frac{x}{2}\,dx = 2\int_0^{\ln 3} \cosh u\,du = 2\left[\sinh u\right]_0^{\ln 3}$$

$$= 2\left[\sinh(\ln 3) - \sinh(0)\right] = 2\left[\frac{e^{\ln 3}-e^{-\ln 3}}{2} - 0\right]$$

$$= 3 - \frac{1}{3} = \frac{8}{3}.$$

P.T.O. $\rightarrow$

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Semester II (052)
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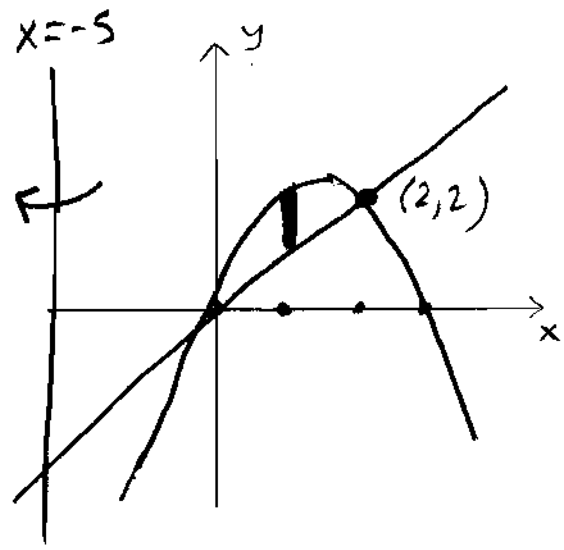
Solve three problem only

1. (5- Points) Use cylindrical shells to **set up** an integral which represents the volume of the solid generated by revolving the area enclosed by the graphs of $y = 3x - x^2$ and $y = x$ about the vertical line $x = -5$.

Points of Intersection $3x - x^2 = x \Rightarrow$

$x^2 - 2x = 0 = x(x-2) \Rightarrow \boxed{x=0, y=0}$, $\boxed{x=2, y=2}$



$$dV = \underbrace{2\pi(x+5)}_{\text{ave. radius}} \; \underbrace{(3x - x^2 - x)}_{\text{height}} \; \underbrace{dx}_{\text{thickness}}$$

$$\Rightarrow V = \int_0^2 2\pi(x+5)(2x - x^2)\,dx$$

2. (5- Points) Find the exact arc length of the parametric curve $x = e^{2t}\cos 3t,\ y = e^{2t}\sin 3t,\ 0 \le t \le \pi/6$.

$$dL = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$\left(\frac{dx}{dt}\right)^2 = \left(2e^{2t}\cos 3t - 3e^{2t}\sin 3t\right)^2 = \left(4e^{4t}\cos^2 3t - 12e^{2t}\cos 3t \sin 3t + 9e^{4t}\sin^2 3t\right)$$

$$\left(\frac{dy}{dt}\right)^2 = \left(2e^{2t}\sin 3t + 3e^{2t}\cos 3t\right)^2 = 4e^{4t}\sin^2 3t + 12e^{2t}\sin 3t\cos 3t + 9e^{4t}\cos^2 3t)$$

$$\Rightarrow \left(\frac{dx}{dy}\right)^2 = 4e^{4t} + 9e^{4t} = 13e^{4t} \Rightarrow$$

$$L = \int_0^{\pi/6} \sqrt{13e^{4t}}\,dt = \sqrt{13}\int_0^{\pi/6} e^{2t}\,dt = \frac{\sqrt{13}}{2}\left[e^{2t}\right]_0^{\pi/6}$$

$$= \frac{\sqrt{13}}{2}\left[e^{\pi/3} - 1\right].$$

P.T.O →

3. (5- Points) find the area of the surface generated by revolving the curve $y=\sqrt{4-x^2}$, $-1 \le x \le 1$ about the x-axis.

$$dS = 2\pi y \, dL \quad , \quad dL = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{-2x}{2\sqrt{4-x^2}} \Rightarrow 1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{x^2}{4-x^2} = \frac{4}{4-x^2}$$

$$\Rightarrow dS = 2\pi\sqrt{4-x^2}\sqrt{\frac{4}{4-x^2}}\, dx = 4\pi\, dx \Rightarrow$$

$$S = \int_{-1}^{1} 4\pi\, dx = 4\pi [x]_{-1}^{1} = 8\pi.$$

4. (5-Points) (a) Evaluate $\lim\limits_{x\to-\infty} \tanh\frac{x}{3}$. $= \lim\limits_{x\to-\infty} \dfrac{e^{x/3} - e^{-x/3}}{e^{x/3} + e^{-x/3}}$

$$= \lim_{x\to-\infty} \frac{e^{2x/3}-1}{e^{2x/3}+1} = -1$$

(b) Evaluate $\int_0^{\ln 8} \cosh\frac{x}{3}\, dx$. [write your answer in simplest form]

Put $u = \frac{x}{3} \Rightarrow du = \frac{1}{3}dx$, $x=0 \Rightarrow u=0$, $x=\ln 8 \Rightarrow u=\ln 2$

$$\Rightarrow \int_0^{\ln 8} \cosh\frac{x}{3}\, dx = 3\int_0^{\ln 2} \cosh u\, du = 3\left[\sinh u\right]_0^{\ln 2}$$

$$= 3\left[\sinh(\ln 2) - \sinh(0)\right] = 3\left[\frac{e^{\ln 2} - e^{-\ln 2}}{2} - 0\right]$$

$$= \frac{3}{2}\left[2-\frac{1}{2}\right] = \frac{9}{4}.$$

P.T.O