KFUPM | Semester II (052) | Name: KEY | Serial:
Math 102-4, 8 | Quiz #4 | ID#: | Sec #:

Solve three problems only

1. (5- Points) Evaluate $\int x \tan^{-1}\frac{x}{2}\,dx$

Put $u = \tan^{-1}\frac{x}{2}$ and $dv = x\,dx$

$\Rightarrow du = \dfrac{1/2}{1+\frac{x^2}{4}}\,dx$ and $v = \frac{1}{2}x^2$

$$\Rightarrow \int x\tan^{-1}\frac{x}{2} = \frac{1}{2}x^2\tan^{-1}\frac{x}{2} - \int \frac{x^2}{x^2+4}\,dx$$

$$= \frac{1}{2}x^2\tan^{-1}\frac{x}{2} - \int\left(1 - \frac{4}{x^2+4}\right)dx$$

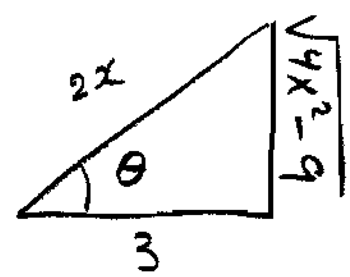
$$= \frac{1}{2}x^2\tan^{-1}\frac{x}{2} - x + 2\tan^{-1}\left(\frac{x}{2}\right) + C.$$

2. Evaluate $\int \frac{dx}{x^2\sqrt{4x^2-9}} = I$

Put $x = \frac{3}{2}\sec\theta \Rightarrow dx = \frac{3}{2}\sec\theta\tan\theta\,d\theta,$

$\sqrt{4x^2-9} = 3\tan\theta, \quad \theta = \sec^{-1}\frac{2x}{3}$

$$\Rightarrow I = \int \frac{\frac{3}{2}\sec\theta\tan\theta\,d\theta}{\left(\frac{9}{4}\sec^2\theta\right)(3\tan\theta)}$$



$$= \frac{2}{9}\int\frac{1}{\sec\theta}\,d\theta = \frac{2}{9}\int\cos\theta\,d\theta$$

$$= \frac{2}{9}\sin\theta + C$$

$$= \frac{1}{9}\frac{\sqrt{4x^2-9}}{x} + C$$

P.T.O →

• 3. (5-Points) Evaluate $\int \frac{\tan^3 x}{\sqrt{\sec x}} dx = \int \frac{\tan^2 x}{\sqrt{\sec x}} \frac{(\tan x \sec x \, dx)}{\sec x}$

$= \int (\sec^2 x - 1)(\sec x)^{-3/2} (\tan x \sec x \, dx)$

Put $u = \sec x \Rightarrow du = \sec x \tan x \, dx \Rightarrow$

$\int \frac{\tan^3 x}{\sqrt{\sec x}} dx = \int (u^{1/2} - u^{-3/2}) \, du$

$= \frac{2}{3} u^{3/2} + 2u^{-1/2} + C$

$= \frac{2}{3} (\sec x)^{3/2} + 2(\sec x)^{-1/2} + C.$

4. (5- Points) Evaluate $\int (2x+3)^2 e^{2x} dx$

In this problem we can use integration by parts directly or tabular integration by parts.

$$\begin{array}{lcl} (2x+3)^2 & & e^{2x} \\ & + & \\ 4(2x+3) & & \frac{1}{2} e^{2x} \\ & - & \\ 8 & & \frac{1}{4} e^{2x} \\ & + & \\ 0 & & \frac{1}{8} e^{2x} \end{array}$$

Thus $\int (2x+3)^2 e^{2x} dx = \frac{1}{2}(2x+3)^2 e^{2x} - (2x+3) e^{2x} + e^{2x} + C.$

P.T.O.

KFUPM
Math 102-4, 8

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Quiz #4

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Solve three problems only

1. (5- Points) Evaluate $\int x \tan^{-1}\frac{x}{3}\,dx$

Put $u = \tan^{-1}\frac{x}{3}$ and $v\,dv = x\,dx$

$$\Rightarrow \quad du = \frac{1/3}{1+\frac{x^2}{9}}\,dx \quad \text{and} \quad v = \frac{1}{2}x^2$$

$$\Rightarrow \int x \tan^{-1} x\,dx = \frac{1}{2}x^2 \tan^{-1}\frac{x}{3} - \frac{3}{2}\int \frac{x^2}{9+x^2}\,dx$$

$$= \frac{1}{2}x^2 \tan^{-1}\frac{x}{3} - \frac{3}{2}\int\left[1 - \frac{9}{9+x^2}\right]dx$$

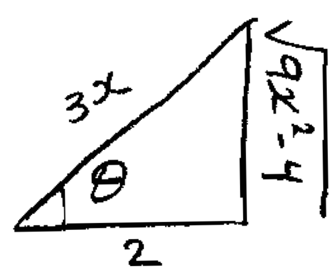
$$= \frac{1}{2}x^2 \tan^{-1}\frac{x}{3} - \frac{3}{2}x + \frac{9}{2}\tan^{-1}\frac{x}{3} + C$$

2. Evaluate $\int \frac{dx}{x^2\sqrt{9x^2-4}} = I$

Put $x = \frac{2}{3}\sec\theta \Rightarrow dx = \frac{2}{3}\sec\theta\tan\theta\,d\theta$,

$\sqrt{9x^2-4} = 2\tan\theta$, $\theta = \sec^{-1}\frac{3x}{2}$

$$I = \int \frac{\frac{2}{3}\sec\theta\tan\theta\,d\theta}{\left(\frac{4}{9}\sec^2\theta\right)(2\tan\theta)}$$



$$= \frac{3}{4}\int \frac{1}{\sec\theta}\,d\theta = \frac{3}{4}\int \cos\theta\,d\theta$$

$$= \frac{3}{4}\sin\theta + C$$

$$= \frac{1}{4}\frac{\sqrt{9x^2-4}}{x} + C$$

3. (5-Points) Evaluate $\int \frac{\tan^3 x}{\sqrt[3]{\sec x}} dx$ $= \int \frac{\tan^2 x \;(\tan x \sec x \; dx)}{\sqrt[3]{\sec x} \;\; \sec x}$

$= \int (\sec^2 x - 1)(\sec x)^{-4/3} (\tan x \sec x \; dx)$

Put $u = \sec x \Rightarrow du = \sec x \tan x \, dx \Longrightarrow$

$\int \frac{\tan^3 x}{\sqrt[3]{\sec x}} dx = \int (u^{2/3} - u^{-4/3}) \, du$

$= \frac{3}{5} u^{5/3} + 3 u^{-1/3} + C$

$= \frac{3}{5} (\sec x)^{5/3} + 3 (\sec x)^{-1/3} + C .$

4. (5- Points) Evaluate $\int (3x+2)^2 e^{3x} dx.$

In this problem we can use integration by parts directly or tabular integration by parts

$(3x+2)^2$		e^{3x}
	$+$	
$6(3x+2)$		$\frac{1}{3} e^{3x}$
	$-$	
18		$\frac{1}{9} e^{3x}$
	$+$	
0		$\frac{1}{27} e^{3x}$

Thus $\int (3x+2)^2 e^{3x} dx = \frac{1}{3} (2x+3)^2 e^{3x} - \frac{2}{3} (3x+2) e^{3x}$

$+ \frac{2}{3} e^{3x} + C .$