

~~Problem 102-15~~

1. If the region enclosed by  $y = \sin x$  and the  $x$ -axis from  $x = 0$  to  $x = \pi$  is revolved about the vertical line  $x = -1$ , then the volume of the solid generated is equal to

- (a)  $2\pi(\pi - 4)$
- (b)  $2\pi^2$
- (c)  $2\pi(\pi - 1)$
- (d)  $2\pi(\pi + 2)$
- (e)  $2\pi(\pi + 4)$

2. If  $a_k = \frac{(k!)^2(3^k)}{(2k)!}$  and  $\lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} = L$ , then the series  $\sum_{k=1}^{\infty} a_k$

- (a) diverges because  $L = \frac{4}{3} > 1$
- (b) diverges because  $L = 12 > 1$
- (c) converges because  $L = \frac{3}{4} < 1$
- (d) converges because  $L = \frac{1}{12} < 1$
- (e) may converge or diverge because  $L = 1$

3.  $\int \frac{3x^2 + x + 4}{x^3 + 4x} dx =$

(a)  $\ln |x(x^2 + 4)| + \frac{3}{2} \tan^{-1} \left( \frac{x}{2} \right) + c$

(b)  $\ln |x\sqrt{x^2 + 4}| + c$

(c)  $\ln |x\sqrt{x^2 + 4}| + \frac{3}{2} \tan^{-1} \left( \frac{x}{2} \right) + c$

(d)  $\ln |x(x^2 + 4)| + \frac{1}{2} \tan^{-1} \left( \frac{x}{2} \right) + c$

(e)  $\ln |x\sqrt[3]{x^2 + 4}| + \frac{1}{2} \tan^{-1} \left( \frac{x}{2} \right) + c$

4. If  $\{S_n\}_{n=1}^{+\infty}$  is the sequence of partial sums of the series  $\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots$ , then  $S_n$  is equal to

(a)  $\frac{n+1}{6(2n+3)}$

(b)  $\frac{2}{(2n+1)(2n+3)}$

(c)  $\frac{n}{3(2n+3)}$

(d)  $\frac{6n}{2n+3}$

(e)  $\frac{n}{2n+1}$

5.  $\int_0^{\ln 2} \frac{\sinh x}{1 + \cosh x} dx =$
- (a)  $\ln(9/8)$
  - (b)  $\ln(4/5)$
  - (c)  $\ln(12/7)$
  - (d)  $\ln(15/8)$
  - (e)  $\ln 36$

6. If  $a_k = \frac{19}{\sqrt[3]{27k^2 + 5k}}$ ,  $b_k = \frac{1}{k^{2/3}}$ , and  $\lim_{k \rightarrow \infty} \frac{a_k}{b_k} = L$ , then the series  $\sum_{k=1}^{\infty} a_k$
- (a) converges because  $L > 1$  and finite
  - (b) diverges because  $L < 1$  and finite
  - (c) converges because  $L < 1$  and finite
  - (d) diverges because  $L = +\infty$
  - (e) diverges because  $L > 1$  and finite

7. If the integral test is used in the interval  $[0, \infty)$  for the series  $\sum_{k=0}^{\infty} \frac{1}{1+9k^2}$ , then the value of the integral is
- (a)  $+\infty$ , and the series diverges
  - (b)  $\frac{\pi}{9}$ , and the series diverges
  - (c)  $\frac{\pi}{6}$ , and the series converges
  - (d)  $9\pi$ , and the series converges
  - (e)  $\frac{9\pi}{2}$ , and the series converges

8. The improper integral  $\int_0^e x \ln x \, dx$
- (a) diverges
  - (b) converges and has the value  $e^2$
  - (c) converges and has the value  $4e^2$
  - (d) converges and has the value  $\frac{1}{4} e^2$
  - (e) oscillates between  $e^2$  and  $4e^2$

9. If the area enclosed by the curves  $y^2 = x$  and  $y = x - 2$  is revolved about the line  $x = 4$ , then the volume of the solid generated is equal to

(a)  $\pi \int_0^4 [(4-x)^2 - (2-x)^2] dx$

(b)  $\pi \int_{-1}^2 [(2+y-y^2)^2] dy$

(c)  $2\pi \int_1^4 (4-x)(\sqrt{x}-x+2) dx$

(d)  $2\pi \int_1^4 (4-x)(2-\sqrt{x}+x) dx$

(e)  $\pi \int_{-1}^2 [(4-y^2)^2 - (2-y)^2] dy$

10. Which one of the following series is absolutely convergent?

(a)  $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$

(b)  $\sum_{k=1}^{\infty} \frac{(-1)^k k}{k+1}$

(c)  $\sum_{k=1}^{\infty} \cos(\pi k)$

(d)  $\sum_{k=1}^{\infty} (-1)^k \pi^k$

(e)  $\sum_{k=1}^{\infty} (-1)^k e^{-k}$

11. The sequence  $\left\{ \tan^{-1} n \right\}_{n=1}^{+\infty}$  is
- (a) increasing (not strictly) with an upper bound
  - (b) strictly increasing **without** an upper bound
  - (c) increasing (not strictly) **without** an upper bound
  - (d) strictly increasing with an upper bound
  - (e) neither increasing nor decreasing
12. The radius of convergence  $R$  and the interval of convergence  $I$  of the power series  $\sum_{k=0}^{\infty} (-1)^k \frac{(x-5)^k}{3^k}$  are
- (a)  $R = \frac{5}{3}$  and  $I = (2, 8)$
  - (b)  $R = 3$  and  $I = (2, 8)$
  - (c)  $R = 3$  and  $I = (2, 8]$
  - (d)  $R = \frac{5}{3}$  and  $I = [2, 8)$
  - (e)  $R = 3$  and  $I = [2, 8)$

13. The series  $\sum_{k=1}^{\infty} 2^{3k} \cdot 3^{1-2k}$

- (a) converges and has the sum  $8/9$
- (b) converges and has the sum  $3/4$
- (c) oscillates between 2 and 3
- (d) diverges
- (e) converges and has the sum 24

14. The sequence  $\frac{\ln 3}{\ln 5}, \frac{\ln 6}{\ln 7}, \frac{\ln 9}{\ln 9}, \frac{\ln 12}{\ln 11}, \dots$

- (a) converges to  $3/2$
- (b) converges to 1
- (c) converges to  $2/3$
- (d) diverges
- (e) oscillates between 2 and 3

15. If the portion of the curve  $y = 2\sqrt{x}$  between  $x = 0$  and  $x = 8$  is revolved about the  $x$ -axis, then the area of the resulting surface is equal to
- (a)  $104\pi$
  - (b)  $\frac{208\pi}{3}$
  - (c)  $\frac{215\pi}{3}$
  - (d)  $\frac{103\pi}{3}$
  - (e)  $16\pi$

16. The value of  $\lim_{\max \Delta x_k \rightarrow 0} \sum_{k=1}^n (\tan x_k^*)^2 (\sec x_k^*)^4 \Delta x_k$  over the interval  $\left[0, \frac{\pi}{4}\right]$  is equal to

- (a)  $\frac{2}{15}$
- (b)  $\frac{11}{15}$
- (c)  $\frac{8}{15}$
- (d)  $\frac{13}{15}$
- (e)  $\frac{14}{15}$