KFUPM SEM II (Term 052) Name: ______________ Serial #: ______

MATH 102-8 Quiz # 1 ID: #: ______________ KEY

1. (3-points) Evaluate $\int \frac{\cos(1-4e^{-x})}{e^x}\,dx. = I$

Let $u = 1-4e^{-x} \Rightarrow du = 4e^{-x}dx \Rightarrow \frac{1}{4}du = \frac{dx}{e^x}$

$\Rightarrow I = \int (\cos u)\left(\frac{1}{4}du\right) = \frac{1}{4}\int \cos u \, du$

$= \frac{1}{4}\sin u + C$

$= \frac{1}{4}\sin(1-4e^{-x}) + C$

2. (3-points) Evaluate $\int \frac{3+5x+5x^3}{1+x^2}\,dx.$

$= \int \frac{3+5x(1+x^2)}{1+x^2}\,dx$

$= \int \left[\frac{3}{1+x^2} + 5x\right] dx$

$= 3\tan^{-1}x + \frac{5}{2}x^2 + C$

3. (5-points) Solve the initial-value problem

$$\frac{dy}{dx} = \frac{3x^2}{\sqrt{1-5x^3}}, \quad y\left(\sqrt[3]{-\frac{3}{5}}\right) = \frac{5}{3}.$$

Let $u = 1-5x^3 \Rightarrow du = -15x^2\,dx \Rightarrow$

$$y = \int \frac{3x^2}{\sqrt{1-5x^3}}\,dx = \int \frac{-\frac{3}{15}\,du}{\sqrt{u}}$$

$$= -\frac{1}{5}\int u^{-1/2}\,du = -\frac{2}{5}u^{1/2} + C$$

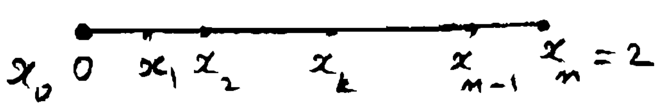
$$= -\frac{2}{5}\sqrt{1-5x^3} + C$$

$$y\left(\sqrt[3]{-\frac{3}{5}}\right) = \frac{5}{3} \Rightarrow \frac{5}{3} = -\frac{2}{5}\sqrt{4} + C = -\frac{4}{5} + C$$

$$C = \frac{5}{3} + \frac{4}{5} = \frac{37}{15} \Rightarrow$$

$$y = -\frac{2}{5}\sqrt{1-5x^3} + \frac{37}{15}$$

4. (4-points) Find the area under the curve $f(x) = 14 - x^3$ from $x = 0$ to $x = 2$, with x_k^* as the right endpoint of each subinterval. $\left[\text{Hint: } \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2\right]$.



$\boxed{\Delta x = \frac{2}{n}}$

$x_0 = 0,\ x_1 = \frac{2}{n},\ x_2 = 2\left(\frac{2}{n}\right),\ x_3 = 3\left(\frac{2}{n}\right), \ldots,\ x_k = k\left(\frac{2}{n}\right), \ldots$

The area $= \lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} f(x_k^*)\,\Delta x = \lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \left[14 - k^3\left(\frac{8}{n^3}\right)\right]\frac{2}{n}$

$$= \lim_{n\to\infty}\left[\frac{2}{n}\sum_{k=1}^{n} 14 - \frac{16}{n^4}\sum_{k=1}^{n} k^3\right]$$

$$= \lim_{n\to\infty}\left[\frac{2}{n}(14n) - \frac{16}{n^4}\left(\frac{n(n+1)}{2}\right)^2\right] \overset{?}{=} 28 - 4 = 24$$