KFUPM SEM II (Term 052) Name: ____________ Serial #: ________

MATH 102-4 Quiz # 1 ID: #: ____________ KEY

1. (3-points) Evaluate $\int \frac{\sin(1+5e^{-x})}{e^x}\,dx.$ $= I$

Let $u = 1+5e^{-x} \Rightarrow du = -5e^{-x}dx$

$\Rightarrow \frac{1}{e^x}dx = -\frac{1}{5}du \Rightarrow$

$I = \int \sin u \left(-\frac{1}{5}du\right) = -\frac{1}{5}\int \sin u\, du$

$= \frac{1}{5}\cos u + C$

$= \frac{1}{5}\cos(1+5e^{-x}) + C$

2. (3-points) Evaluate $\int \frac{3x^3+3x+5}{x^2+1}\,dx.$

$= \int \frac{3x(x^2+1)+5}{x^2+1}\,dx$

$= \int \left[3x + \frac{5}{x^2+1}\right]dx$

$= 3\int x\,dx + 5\int \frac{1}{1+x^2}\,dx$

$= \frac{3}{2}x^2 + 5\tan^{-1}x + C$

3. (5-points) Solve the initial-value problem

$$\frac{dy}{dx}=\frac{5x^2}{\sqrt{1-2x^3}}\,,\qquad y\left(\sqrt[3]{-\frac{3}{2}}\right)=\frac{2}{3}\,.$$

$$\Rightarrow\quad y=\int \frac{5x^2}{\sqrt{1-2x^3}}\,dx$$

Let $u=1-2x^3 \Rightarrow du=-6x^2\,dx \Rightarrow$

$$y=-\frac{5}{6}\int u^{-1/2}\,du=-\frac{5}{6}\left(2u^{1/2}\right)+C$$

$$\Rightarrow\quad y=-\frac{5}{3}\sqrt{1-2x^3}+C$$

$$y\left(\sqrt[3]{-3/2}\right)=\frac{2}{3}\Rightarrow \frac{2}{3}=-\frac{5}{3}\sqrt{1+3}+C\Rightarrow$$

$$\frac{2}{3}=-\frac{10}{3}+C\Rightarrow C=\frac{12}{3}=4\Rightarrow$$

$$y=-\frac{5}{3}\sqrt{1-2x^3}+4$$

4. (4-points) Find the area under the curve $f(x)=12-x^3$ from $x=0$ to $x=2$, with x_k^* as the right endpoint of each subinterval. $\left[\text{Hint: } \sum_{k=1}^{n}k^3=\left[\frac{n(n+1)}{2}\right]^2\right]$.

$x_0=0 \quad x_1 \quad x_2 \quad x_3 \quad x_k \quad x_{n-1} \quad 2=x_n$, $\boxed{\Delta x=\frac{2}{n}}$

$x_0=0,\ x_1=\frac{2}{n},\ x_2=2\left(\frac{2}{n}\right),\ \dots,\ x_k=k\left(\frac{2}{n}\right),\ \dots$

$$\text{Area}=\lim_{n\to\infty}\sum_{k=1}^{n}f(x_k^*)\,\Delta x=\lim_{n\to\infty}\sum_{k=1}^{n}\left[12-k^3\left(\frac{8}{n^3}\right)\right]\left(\frac{2}{n}\right)$$

$$=\lim_{n\to\infty}\left[\frac{2}{n}\sum_{k=1}^{n}12-\frac{16}{n^4}\sum_{k=1}^{n}k^3\right]$$

$$=\lim_{n\to\infty}\left[\frac{2}{n}(12n)-\frac{16}{n^4}\left(\frac{(n+1)n}{2}\right)^2\right]\stackrel{?}{=}24-4=20$$