MATH 260-2-4 (Term 051) Practice Problems
Sections 1.1, 1.2 and 1.4

1. In Problem (a) and (b), state the order of the given DE

   (a) $x\dfrac{d^2y}{dx^2} - \left(\dfrac{dy}{dx}\right)^4 + y = 0.$

   (b) $\dfrac{d^3y}{dx^3} = \sqrt{\left(\dfrac{d^5y}{dx^5}\right)^2 + \left(\dfrac{dy}{dx}\right)^{10}}.$

2. In Problems (a) and (b), verify that the indicated function is a solution of the given DE:

   (a) $y'' - 4y' + 13y = 0, \quad y = e^{2x}\cos 3x.$

   (b) $y'' + y = \tan x, \quad y = -(\cos x)\ln(\sec x + \tan x).$

3. Find values of the constant m so that $y = x^m$ is a solution of the DE

$$x^2y'' - 7xy' + 15y = 0.$$

4. In Problems (a) and (b), find a mathematical model which represents the given family of curves

   (a) $y = \tan(x + c)$, where c is an arbitrary constant.

   (b) $y = c_1e^x + c_2e^{-x}$, where c_1 and c_2 are arbitrary constants.

5. Given the fact that: the population of a country grows at a certain time is proportional to the total population of the country at that time. Find the Mathematical Model in terms of a DE which represents this situation if the population P at time $t = 0$ is P_0. Then solve the DE if $P(t_1) = P_1$.

6. Radium decomposes at a rate proportional to the quantity of radium present. Suppose that it is found that in 25 years, 1.1% of certain quantity of radium decomposed. Determine approximately how long it will take for one-half the original amount of radium to decompose (This is called the half-life problem).

7. Solve $\dfrac{dy}{dx} = y^2 - 4.$

8. Solve the initial value problem

$$(e^{2y} - y)\cos x\frac{dy}{dx} = e^y\sin 2x, \quad y(0) = 0.$$

9. Solve $\dfrac{dy}{dx} = \dfrac{xy + 3x - y - 3}{xy - 2x + 4y - 8}.$

10. Solve the initial value problem

$$\sqrt{1-y^2}\,dx - \sqrt{1-x^2}\,dy = 0, \quad y(0) = \frac{\sqrt{2}}{2}.$$