KFUPM SEM II (Term 072) Name: KEY Serial #: ______

MATH 260-1-2 Quiz # 2 ID: #: ______

1. (5-points) Find a sequence of elementary matrices whose product is the inverse of the matrix $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$. **Verify your answer.**

$$\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ \boxed{4} & 3 & 0 & 1 \end{array}\right] \xrightarrow{E_1} \left[\begin{array}{cc|cc} 2 & \boxed{1} & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right] \qquad E_1 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\xrightarrow{E_2} \left[\begin{array}{cc|cc} 2 & 0 & 3 & -1 \\ 0 & 1 & -2 & 1 \end{array}\right] \qquad E_2 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\xrightarrow{E_3} \left[\begin{array}{cc|cc} 1 & 0 & 3/2 & -1/2 \\ 0 & 1 & -2 & 1 \end{array}\right] \qquad E_3 = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3/2 & -1/2 \\ -2 & 1 \end{bmatrix} \quad \underline{\underline{\text{Verification}}} \quad E_3 E_1 E_2 = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 3/2 & -1/2 \\ -2 & 1 \end{bmatrix} \checkmark$$

2. (4-points) Use Cramer's Rule to find x_3 where

$$\begin{aligned} x_1 + x_2 - x_3 &= 2 \\ 3x_1 - x_2 + x_3 &= -1 \\ x_1 \quad\quad + x_3 &= 0 \end{aligned} \iff \underbrace{\begin{bmatrix} 1 & 1 & -1 \\ 3 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}}_{B}$$

$$|A| = \begin{vmatrix} 1 & 1 & -1 \\ 3 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix} \underset{\text{row}}{\overset{\text{3rd}}{=}} (1-1) + 0 + (-1-3) = -4$$

$$|A_3| = \begin{vmatrix} 1 & 1 & 2 \\ 3 & -1 & -1 \\ 1 & 0 & 0 \end{vmatrix} \underset{\text{row}}{\overset{\text{3rd}}{=}} (-1+2) = 1$$

$$\Rightarrow x_3 = \frac{|A_3|}{|A|} = -\frac{1}{4}.$$

3. (6-points) Given $A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & -1 & 1 \\ 0 & 3 & 2 \end{bmatrix}$.

(a) Use **determinants** to show that A is nonsingular.

(b) Find adj A.

(c) Use (a) and (b) to find A^{-1}.

(a) $|A| = \begin{vmatrix} 1 & 0 & 3 \\ 1 & -1 & 1 \\ 0 & 3 & 2 \end{vmatrix} = (-2-3) + 0 + 3(3-0) = 4 \neq 0$

$\Rightarrow$ A is nonsingular

(b) $\text{Cof } A = \begin{bmatrix} -5 & -2 & 3 \\ 9 & 2 & -3 \\ 3 & 2 & -1 \end{bmatrix} \Rightarrow$

$\text{adj } A = (\text{Cof } A)^T = \begin{bmatrix} -5 & 9 & 3 \\ -2 & 2 & 2 \\ 3 & -3 & -1 \end{bmatrix}$

(c) $A^{-1} = \frac{1}{|A|} \text{ adj } A$

$= \begin{bmatrix} -5/4 & 9/4 & 3/4 \\ -1/2 & 1/2 & 1/2 \\ 3/4 & -3/4 & -1/4 \end{bmatrix}$

Check: $AA^{-1} = \begin{bmatrix} 1 & 0 & 3 \\ 1 & -1 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} -5/4 & 9/4 & 3/4 \\ -1/2 & 1/2 & 1/2 \\ 3/4 & -3/4 & -1/4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ✓

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1. (5-points) Find a sequence of elementary matrices whose product is the inverse of the matrix $\begin{bmatrix} 3 & 2 \\ 6 & 5 \end{bmatrix}$. **Verify your answer.**

$$\left[\begin{array}{cc|cc} 3 & 2 & 1 & 0 \\ \boxed{6} & 5 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cc|cc} 3 & \boxed{2} & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right] \quad E_1 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\longrightarrow \left[\begin{array}{cc|cc} 3 & 0 & 5 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right] \Longrightarrow E_2 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

$$\longrightarrow \left[\begin{array}{cc|cc} 1 & 0 & -5/3 & -2/3 \\ 0 & 1 & -3 & 1 \end{array}\right] \Longrightarrow E_3 = \begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Longrightarrow A^{-1} = E_3 E_2 E_1 = \begin{bmatrix} -5/3 & -2/3 \\ -3 & 1 \end{bmatrix}$$

Check: $E_3 E_2 E_1 = \begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 5/3 & -2/3 \\ -2 & 1 \end{bmatrix} = A^{-1}$$

2. (4-points) Use Cramer's Rule to find x_3 where

$$\begin{aligned} x_1 + \quad + x_3 &= 2 \\ x_1 + x_2 - x_3 &= -1 \\ 3x_1 - x_2 + x_3 &= 0 \end{aligned} \iff \underbrace{\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}}_{B}$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ 3 & -1 & 1 \end{vmatrix} \overset{1st}{\underset{row}{=}} (0) + (0) - 4 = -4$$

$$|A_3| = \begin{vmatrix} 1 & 0 & 2 \\ 1 & 1 & -1 \\ 3 & -1 & 0 \end{vmatrix} = -1 + 0 + 2(-4) = -9$$

$$\Longrightarrow x_3 = \frac{|A|}{|A_3|} = \frac{9}{4}.$$

3. (6-points) Given $A = \begin{bmatrix} 3 & 0 & 3 \\ 1 & 1 & -1 \\ 2 & 2 & 0 \end{bmatrix}$.

(a) Use **determinants** to show that A is nonsingular.

(b) Find adj A.

(c) Use (a) and (b) to find A^{-1}.

(a) $|A| = \begin{vmatrix} 3 & 0 & 3 \\ 1 & 1 & -1 \\ 2 & 2 & 0 \end{vmatrix} = 3(2) + 0 + 3(0) = 6 \neq 0$

$\Rightarrow$ A is nonsingular

(b) $\text{Cof } A = \begin{bmatrix} 2 & -2 & 0 \\ 6 & -6 & -6 \\ -3 & 6 & 3 \end{bmatrix}$

$\Rightarrow \text{adj } A = (\text{Cof } A)^T = \begin{bmatrix} 2 & 6 & -3 \\ -2 & -6 & 6 \\ 0 & -6 & 3 \end{bmatrix}$

(c) $A^{-1} = \frac{1}{|A|} \text{adj } A = \begin{bmatrix} 1/3 & 1 & -1/2 \\ -1/3 & -1 & 1 \\ 0 & -1 & 1/2 \end{bmatrix}$

Check: $A A^{-1} = \begin{bmatrix} 3 & 0 & 3 \\ 1 & 1 & -1 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1/3 & 1 & -1/2 \\ -1/3 & -1 & 1 \\ 0 & -1 & 1/2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ✓