KFUPM SEM II (Term 072) Name: ______ Serial #: ______

MATH 260-1-3 Quiz # 1 ID: #: KEY

1. (7-points) Solve $xy\frac{dy}{dx} = 1 + y - x - xy.$

$$\Rightarrow \quad xy\,\frac{dy}{dx} = (1+y) - x(1+y)$$
$$= (1+y)(1-x)$$

$$\Rightarrow \quad \frac{y}{1+y}\,dy = \frac{1-x}{x}\,dx$$

$$\Rightarrow \quad \int \frac{y}{1+y}\,dy = \int \frac{1-x}{x}\,dx + C$$

$$\Rightarrow \quad \int \left(1 - \frac{1}{1+y}\right) dy = \int \left(\frac{1}{x} - 1\right) dx + C \Rightarrow$$

the G.S. is $y - \ln|1+y| = \ln|x| - x + C$.

2. (8-points) Solve the IVP

$$(x^2+1)\frac{dy}{dx} - 2xy = 8x^2(x^2+1)\ ;\quad y(1)=0.$$

It is a linear first-order DE $\Longrightarrow$

$$\frac{dy}{dx} - \frac{2x}{x^2+1}\,y = 8x^2$$

I.F. $h(x) = e^{-\int \frac{2x}{x^2+1}dx} = e^{-\ln(x^2+1)} = \frac{1}{x^2+1}$

$$\Longrightarrow \quad \frac{1}{x^2+1}\,y = \int \frac{1}{x^2+1}\cdot 8x^2\,dx + C$$

$$= 8\int \left[1 - \frac{1}{1+x^2}\right] dx + C$$

$$= 8\left[x - \tan^{-1} x\right] + C$$

$$\Longrightarrow \quad y = 8x(x^2+1) - 8(x^2+1)\tan^{-1}x + C(x^2+1)$$

which is the G.S.

$y(1)=0 \Rightarrow 0 = 16 - 16\tan^{-1}1 + 2C$

$\Rightarrow 2C = -16 + 16\,\frac{\pi}{4} = -16 + 4\pi$

$\Rightarrow C = -8 + 2\pi \Longrightarrow$

The solution of the given IVP is

$$y = (x^2+1)\left[8x - 8\tan^{-1}x + 2\pi - 8\right]$$

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1. (7-points) Solve $(x - y - xy + 1)\dfrac{dy}{dx} = xy.$

$$\Rightarrow \left[(x+1) - y(1+x)\right]\frac{dy}{dx} = xy$$

$$\Rightarrow (x+1)(1-y)\frac{dy}{dx} = xy$$

$$\Rightarrow \frac{1-y}{y}\,dy = \frac{x}{1+x}\,dx$$

$$\Rightarrow \int \frac{1-y}{y}\,dy = \int \frac{x}{1+x}\,dx + C$$

$$\Rightarrow \int \left[\frac{1}{y} - 1\right] dy = \int \left[1 - \frac{1}{1+x}\right] dx + C$$

$\Rightarrow$ the G.S. is

$$\ln|y| - y = x - \ln|1+x| + C.$$

2. (8-points) Solve the IVP

$$(x^2+1)\frac{dy}{dx} + 2xy = \frac{4x^2}{x^2+1}\ ; \quad y(1) = 0.$$

It in a linear first-order DE $\Longrightarrow$

$$\frac{dy}{dx} + \frac{2x}{x^2+1}\, y = \frac{4x^2}{(x^2+1)^2}$$

$$\text{I.F. } h(x) = e^{\int \frac{2x}{x^2+1}dx} = e^{\ln(x^2+1)} = x^2+1$$

$$\Longrightarrow (x^2+1)\, y = \int (x^2+1)\left(\frac{4x^2}{(x^2+1)^2}\right) dx + C$$

$$= 4\int \frac{x^2}{x^2+1}\, dx + C.$$

$$= 4\int \left[1 - \frac{1}{x^2+1}\right] dx + C$$

$$= 4\left[x - \tan^{-1} x\right] + C \Longrightarrow$$

The G.S is $\quad y = \dfrac{4}{x^2+1}\left(x - \tan^{-1} x\right) + \dfrac{C}{x^2+1}$

$$y(1) = 0 \Longrightarrow \quad 0 = \frac{4}{2}\left(1 - \tan^{-1} 1\right) + \frac{C}{2}$$

$$= 2\left(1 - \frac{\pi}{4}\right) + \frac{C}{2}$$

$$\Longrightarrow \quad C = -4\left(1 - \frac{\pi}{4}\right) = \pi - 4 \Longrightarrow$$

The required solution is

$$y = \frac{4}{x^2+1}\left(x - \tan^{-1} x\right) + \frac{\pi - 4}{x^2+1}.$$