

## COINCIDENCES OF LIPSCHITZ-TYPE HYBRID MAPS AND INVARIANT APPROXIMATION

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□ *The aim of this paper is to obtain new coincidence and common fixed point theorems by using Lipschitz-type conditions of hybrid maps (not necessarily continuous) on a metric space. As applications, we demonstrate the existence of common fixed points from the set of best approximations. Our work sets analogues, unifies and improves various known results existing in the literature.*

**Keywords** Best approximation; Coincidence point; Common fixed point; Eigenvalue; Lipschitz condition; Metric space; Weak commutativity; Weakly compatible maps.

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### 1. INTRODUCTION

Common fixed point theorems for families of commuting contraction maps have been a popular area of research (see, e.g., Al-Thagafi [2], and Belluce and Kirk [4]). In 1982, Sessa [18] introduced the concept of weakly commuting maps to generalize commutativity. Jungck [10] generalized weak commutativity to the notion of compatible maps. In 1996, Jungck [11] further weakened compatibility to the concept of weak compatibility. Since then, many interesting fixed point theorems of compatible and weakly compatible maps under contractive conditions have

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