ON LOCALLY SOLID TOPOLOGICAL LATTICE GROUPS

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Abstract. Let (G, τ) be a commutative Hausdorff locally solid lattice group. In this paper we prove the following:
 (1) If (G, τ) has the A(iii)-property, then its completion $(\widehat{G}, \hat{\tau})$ is an order-complete locally solid lattice group.
 (2) If G is order-complete and τ has the Fatou property, then the order intervals of G are τ-complete.
 (3) If (G, τ) has the Fatou property, then G is order-dense in $\widehat{G}$ and $(\widehat{G}, \hat{\tau})$ has the Fatou property.
 (4) The order-bound topology on any commutative lattice group is the finest locally solid topology on it.
 As an application, a version of the Nikodym boundedness theorem for set functions with values in a class of locally solid topological groups is established.

Keywords: topological completion, locally solid ℓ-group, topological continuity, Fatou property, order-bound topology

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1. Introduction

The theory of topological Riesz spaces is very rich, and vector measures with values in these spaces and order-complete Riesz spaces have been extensively studied (for example, see Aliprantis [1], Fremlin [12], Schmidt [18] and Swartz [19]). In recent years, contributions to the theory of topological groups have been made by Comfort et. al [8–9], Bonales [6] and Raczkowski [17]; in particular, they have studied totally bounded group topologies, Bohr topology and the relevance to locally convex spaces of the celebrated theorem of Pontryagin-Van Kampen which states that every locally compact Abelian group satisfies group duality (see [6], p. 76 for the details). Thereby, the school of mathematicians led by W. W. Comfort has generated tremendous activities in this area of investigations. Topological Riesz groups and their special case,