ON LOCALLY SOLID TOPOLOGICAL LATTICE GROUPS

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(Received August 31, 2005)

Abstract. Let \((G, \tau)\) be a commutative Hausdorff locally solid lattice group. In this paper we prove the following:

1. If \((G, \tau)\) has the \(A(iii)\)-property, then its completion \((\hat{G}, \hat{\tau})\) is an order-complete locally solid lattice group.
2. If \(G\) is order-complete and \(\tau\) has the Fatou property, then the order intervals of \(G\) are \(\tau\)-complete.
3. If \((G, \tau)\) has the Fatou property, then \(G\) is order-dense in \(\hat{G}\) and \((\hat{G}, \hat{\tau})\) has the Fatou property.
4. The order-bound topology on any commutative lattice group is the finest locally solid topology on it.

As an application, a version of the Nikodym boundedness theorem for set functions with values in a class of locally solid topological groups is established.

Keywords: topological completion, locally solid \(\ell\)-group, topological continuity, Fatou property, order-bound topology

MSC 2000: 46A40, 54H11, 28B15

1. Introduction

The theory of topological Riesz spaces is very rich, and vector measures with values in these spaces and order-complete Riesz spaces have been extensively studied (for example, see Aliprantis [1], Fremlin [12], Schmidt [18] and Swartz [19]). In recent years, contributions to the theory of topological groups have been made by Comfort et. al [8–9], Bonales [6] and Raczkowski [17]; in particular, they have studied totally bounded group topologies, Bohr topology and the relevance to locally convex spaces of the celebrated theorem of Pontryagin-Van Kampen which states that every locally compact Abelian group satisfies group duality (see [6], p. 76 for the details). Thereby, the school of mathematicians led by W. W. Comfort has generated tremendous activities in this area of investigations. Topological Riesz groups and their special case,