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## Approximating common fixed points of asymptotically nonexpansive maps in uniformly convex Banach spaces

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## Abstract

We introduce three-step iterative schemes with errors for two and three nonexpansive maps and establish weak and strong convergence theorems for these schemes. Mann-type and Ishikawa-type convergence results are included in the analysis of these new iteration schemes. The results presented in this paper substantially improve and extend the results due to [S.H. Khan, H. Fukhar-ud-din, Weak and strong convergence of a scheme with errors for two nonexpansive mappings, Nonlinear Anal. 8 (2005) 1295–1301], [N. Shahzad, Approximating fixed points of non-self nonexpansive mappings in Banach spaces, Nonlinear Anal. 61 (2005) 1031–1039], [W. Takahashi, T. Tamura, Convergence theorems for a pair of nonexpansive mappings, J. Convex Anal. 5 (1995) 45–58], [K.K. Tan, H.K. Xu, Approximating fixed points of nonexpansive mappings by the Ishikawa iteration process, J. Math. Anal. Appl. 178 (1993) 301–308] and [H.F. Senter, W.G. Dotson, Approximating fixed points of nonexpansive mappings. Convex mappings, Proc. Amer. Math. Soc. 44 (1974) 375–380].

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## 1. Introduction

Let *C* be a nonempty convex subset of a real Banach space *E*. A map  $T : C \to C$  is called: (i) nonexpansive if  $||Tx - Ty|| \le ||x - y||$  for all  $x, y \in C$ ; (ii) quasi-nonexpansive if the set F(T) of fixed points of *T* is nonempty and  $||Tx - Ty|| \le ||x - y||$  for all  $x \in C$  and  $y \in F(T)$ .

Das and Debata [1] introduced the following iteration scheme:

$$\begin{cases} x_1 \in C, \\ y_n = (1 - \beta_n) x_n + \beta_n T_2 x_n, \\ x_{n+1} = (1 - \alpha_n) x_n + \alpha_n T_1 y_n, & \text{for all } n \ge 1, \end{cases}$$
(1.1)

where  $T_1, T_2$  are quasi-nonexpansive selfmaps with compact domain and  $\{\alpha_n\}, \{\beta_n\}$  are sequences in [0, 1]. They used the scheme (1.1) to approximate common fixed points of the maps when E is strictly convex. For  $T_1 = T_2$ ,

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