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Approximating common fixed points of asymptotically nonexpansive maps in uniformly convex Banach spaces

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Abstract

We introduce three-step iterative schemes with errors for two and three nonexpansive maps and establish weak and strong convergence theorems for these schemes. Mann-type and Ishikawa-type convergence results are included in the analysis of these new iteration schemes. The results presented in this paper substantially improve and extend the results due to [S.H. Khan, H. Fukhar-ud-din, Weak and strong convergence of a scheme with errors for two nonexpansive mappings, Nonlinear Anal. 8 (2005) 1295–1301], [N. Shahzad, Approximating fixed points of non-self nonexpansive mappings in Banach spaces, Nonlinear Anal. 61 (2005) 1031–1039], [W. Takahashi, T. Tamura, Convergence theorems for a pair of nonexpansive mappings, J. Convex Anal. 5 (1995) 45–58], [K.K. Tan, H.K. Xu, Approximating fixed points of nonexpansive mappings by the Ishikawa iteration process, J. Math. Anal. Appl. 178 (1993) 301–308] and [H.F. Senter, W.G. Dotson, Approximating fixed points of nonexpansive mappings, Proc. Amer. Math. Soc. 44 (1974) 375–380].
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1. Introduction

Let C be a nonempty convex subset of a real Banach space E. A map $T : C \rightarrow C$ is called: (i) nonexpansive if $\|Tx - Ty\| \le \|x - y\|$ for all $x, y \in C$; (ii) quasi-nonexpansive if the set $F(T)$ of fixed points of T is nonempty and $\|Tx - Ty\| \le \|x - y\|$ for all $x \in C$ and $y \in F(T)$.

Das and Debata [1] introduced the following iteration scheme:

$$\begin{cases} x_1 \in C, \\ y_n = (1 - \beta_n)x_n + \beta_n T_2 x_n, \\ x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T_1 y_n, \quad \text{for all } n \ge 1, \end{cases} \tag{1.1}$$

where T_1, T_2 are quasi-nonexpansive selfmaps with compact domain and $\{\alpha_n\}$, $\{\beta_n\}$ are sequences in $[0, 1]$. They used the scheme (1.1) to approximate common fixed points of the maps when E is strictly convex. For $T_1 = T_2$,

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