

# Common fixed points Noor iteration for a finite family of asymptotically quasi-nonexpansive mappings in Banach spaces

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Received 24 January 2006

Available online 14 July 2007

Submitted by R. Curto

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## Abstract

In this paper, we introduce a general iteration scheme for a finite family of asymptotically quasi-nonexpansive mappings. The new iterative scheme includes the modified Mann and Ishikawa iterations, three-step iterative scheme of Xu and Noor and Khan and Takahashi scheme as special cases. Our results are generalizations as well as refinement of several known results in the current literature.

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**Keywords:** Modified Mann and Ishikawa iterations; Asymptotically quasi-nonexpansive mapping; Common fixed point; Weak and strong convergence; Banach space

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## 1. Introduction and preliminaries

Let  $C$  be a nonempty subset of a real Banach space  $X$  and  $T$  a selfmapping of  $C$ . Denote by  $F(T)$ , the set of fixed points of  $T$ . Throughout this paper, we assume that  $F(T) \neq \emptyset$ . The mapping  $T$  is said to be

- (i) nonexpansive if  $\|Tx - Ty\| \leq \|x - y\|$ , for all  $x, y \in C$ ;
- (ii) quasi-nonexpansive if  $\|Tx - p\| \leq \|x - p\|$ , for all  $x \in C$  and  $p \in F(T)$ ;
- (iii) asymptotically nonexpansive if there exists a sequence  $\{u_n\}$  in  $[0, +\infty)$  with  $\lim_{n \rightarrow \infty} u_n = 0$  and  $\|T^n x - T^n y\| \leq (1 + u_n)\|x - y\|$ , for all  $x, y \in C$  and  $n = 1, 2, 3, \dots$ ;
- (iv) asymptotically quasi-nonexpansive if there exists a sequence  $\{u_n\}$  in  $[0, +\infty)$  with  $\lim_{n \rightarrow \infty} u_n = 0$  and  $\|T^n x - p\| \leq (1 + u_n)\|x - p\|$ , for all  $x \in C$ ,  $p \in F(T)$  and  $n = 1, 2, \dots$ ;
- (v) uniformly  $L$ -Lipschitzian if there exists a constant  $L > 0$  such that  $\|T^n x - T^n y\| \leq L\|x - y\|$ , for all  $x, y \in C$  and  $n = 1, 2, 3, \dots$ ;

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