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On a Theorem of Daneš and the Principle of Equicontinuity.

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Sunto. - *Si dimostra un teorema di Daneš in ipotesi più deboli e si sfrutta il risultato per dare una versione del principio di equicontinuità per gruppi topologici.*

In [2] Daneš has proved two theorems, stated as Theorems A and B below, from which the Banach-Steinhaus theorem on the condensation of singularities [1] (see also [3], p. 81) and other results may be derived. In this note we show that Theorem A can be proved under weaker hypothesis. Our result enables us to prove Theorem B under weaker hypothesis; it also enables us to give a version of the principle of equicontinuity for topological groups.

THEOREM A. - *Let G be a commutative topological group such that, for each $x \in G$, there exists an element $x/2$ in G (with $x/2 + x/2 = x$) and the mapping $x \to x/2$ is continuous. Let $\{x_n\}$ be a sequence in G such that $\lim_{n\to\infty} x_n = 0$ and $\{p_n\}$ a sequence of real-valued $\frac{1}{2}$-convex sub-additive functions on G (that is, p_n is sub-additive and $2p_n(x+y) \leq p_n(2x) + p_n(2y)$ for $x, y \in G$, $n = 1, 2, \ldots$). Suppose that there exists a sequence $\{a_k: k = 1, 2, \ldots\}$, with $a_k \to +\infty$ as $k \to \infty$, such that, for all $k, n = 1, 2, \ldots$, the set $B_{k,n} = \{x \in G: p_n(x) \leq a_k\}$ is closed. If $\limsup_n \left(\sup_{x\in U} p_n(x)\right) = +\infty$ for each neighbourhood U of 0 in G, then the set*

$$Z = \left\{z \in G: \limsup_n p_n(x_n + z) = +\infty \text{ or } \limsup_n p_n(x_n - z) = +\infty\right\}$$

is a residual G_δ-set in G.

THEOREM B. - *Let X be a topological vector space, $\{x_n: n = 1, 2, \ldots\}$ a sequence in X such that $\lim_{n\to\infty} x_n = 0$ and $\{p_n: n = 1, 2, \ldots\}$ a sequence*