



# Common Fixed-Point Results in Best Approximation Theory

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**Abstract**—A common fixed-point generalization of the results of Dotson, Tarafdar, and Taylor is obtained which in turn extends a recent theorem by Jungck and Sessa to locally convex spaces. As applications of our work, we improve and unify well-known results on fixed points and common fixed points of best approximation. © 2003 Elsevier Science Ltd. All rights reserved.

**Keywords**—Common fixed point, Best approximation, Nonexpansive map, Weakly commuting maps, Locally convex space.

## 1. INTRODUCTION

Fixed-point theorems have been used in the context of normed spaces and locally convex spaces to find interesting results in approximation theory; for example, the reader is referred to the work of Al-Thagafi [1], Singh [2], and Brosowski [3].

An extension of the Banach contraction principle for weakly commuting maps is obtained by Baskaran and Subrahmanyam [4] in the following form.

**THEOREM A.** *Let  $(X, d)$  be a complete metric space. Let  $f : X \rightarrow X$  be a continuous map and  $g : X \rightarrow X$  a map such that  $g(x) \subseteq f(x)$ ,  $g$  and  $f$  weakly commute and there exists  $k \in (0, 1)$  with  $d(gx, gy) \leq kd(fx, fy)$  for all  $x, y \in X$ . Then  $f$  and  $g$  have a unique common fixed point.*

Using Theorem A, we establish common fixed-point generalization of Theorems 1 and 2 [5], Theorem 1.2 and Corollary 1.2 [6], and Theorem 6 [7]; an application of this result provides extensions of approximation results, namely Theorem 3.2 [1], Theorem 7 [7], Theorem 3 [8], and Theorem 2.1 [9] for the maps which are not necessarily commutative.

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