KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS

STAT 211: BUSINESS STATISTICS I
Semester 063
Major Exam III
Monday 13/8/2007
4:30 pm – 5:30 pm

Please **circle** your section number: **2** (9:20 – 10:20 am) **3** (10:30 – 11:30 am)

Surname: SOLUTION KEY ID#: Serial#:

Question No	Full Marks	Marks Obtained
1	22	
2	18	
3	20	
Total	**60**	

Q1. (8+3+3+8 Points)

An official from the securities commission estimates that 60% of all investment bankers have profited from the use of insider information. If 10 investment bankers are selected at random from the commission's registry, answer the following:

a. The EXACT probability that at least 2 of them have profited from insider information =	0.9983

X : # of bankers profited from insider information in a sample of 10.

$$X : B(10\,,0.6) \rightarrow f(x) = \binom{10}{x} 0.6^{x}\, 0.4^{10-x} \quad , \quad x = 0,1,\ldots,10$$

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - \left[\binom{10}{0} 0.6^{0} 0.4^{10} + \binom{10}{1} 0.6^{1} 0.4^{9}\right]$$

= 1 – (0.0001 + 0.0016)

= 1 – 0.0017 = **0.9983**

b. The EXACT probability that all of the 10 investment bankers have profited from insider information =	0.006

$$P(X = 10) = \binom{10}{10} 0.6^{10} 0.4^{0} = \mathbf{0.006}$$

c. The AVERAGE number of investment bankers that have profited from the use of insider information =	6

$E(X) = np = 10\ (0.6) = \mathbf{6}$ bankers

d. If 100 investment bankers were selected randomly from the commission's registry, then the APPROXIMATE probability that at most half of them have profited from insider information =	0.0207

If $n = 100 \rightarrow np = 100(0.6) = 60 \geq 5$ and $nq = 100(0.4) = 40 \geq 5$

➔ $X : B(100\ , 0.6)$ but $X \sim N(np\ , npq)$

➔ $\bar{p} \sim N(p\ , pq/n)$ but $\bar{p} \sim N(0.6\ , 0.24/100)$

➔ $$P\left(X \leq 50\right) = P\left(\frac{X}{n} \leq \frac{50}{100}\right) = P\left(\bar{P} \leq \frac{1}{2}\right)$$

$$= P\left(\frac{\bar{P} - p}{\sqrt{\frac{pq}{n}}} \leq \frac{\frac{1}{2} - 0.6}{\sqrt{\frac{0.24}{100}}}\right) = P\left(Z \leq \frac{-0.1}{\sqrt{0.24}}\right)$$

$= P(Z < -2.04) = P(Z > 2.04) = ½ – P(0 < Z < 2.04)$

$= ½ – 0.4793 = \mathbf{0.0207}$

Q2. (6+2+6+4 Points)

An advertising executive receives an average of 10 telephone calls each afternoon between **2** and **4** P.M. The calls occur randomly and independently of one another. Find the following:

a. The PROBABILITY that the executive will receive 7 calls BETWEEN 2 AND 3 P.M. on a particular afternoon =	0.1044

X: # of calls in 2 hrs → λ = 10 per 2 hrs = 5 per hr = 1/12 per min.

$X : Po(\lambda t)$ → $P(X = x) = f(x) = \dfrac{e^{-\lambda t} \lambda t^{x}}{x!}$, $x = 0, 1, 2, \ldots$

When $t = ½$ → $\lambda t = (10)(½) = 5$ per hr and

$P(X = 7) = f(7) = \dfrac{e^{-5} 5^{7}}{7!}$ = **0.1044**

b. The VARIANCE of the number of calls that the executive will receive BETWEEN 2 AND 5 P.M. on a particular afternoon =	15

V(X) = $\lambda t = 10 \times \dfrac{3}{2}$ = **15** calls

c. The PROBABILITY that the executive has to wait AT LEAST 15 MINUTES to receive the next call on a particular afternoon =	0.2865

Y : Waiting time in hrs → *Y* : Exp(5) → $f(y) = 5e^{-5y}$, $y \geq 0$

$P(Y \leq a) = 1 - e^{-\lambda a}$ and 15 min = ¼ hr

$P(Y > ¼) = 1 - P(Y < ¼) = 1 - (1 - e^{\frac{-5}{4}}) = e^{\frac{-5}{4}}$ = **0.2865**

d. The EXPECTED time that the executive has to wait until he receives the next TWO calls on a particular afternoon =	24 min 2/5 hr

For ONE call only $E(Y) = 1/\lambda = 1/5$ hr

Hence for TWO calls $E(Y) = 2(1/\lambda)$ = (**2/5**) hr = **24** min

Q3. (3+5+7+5 Points)

Researchers studying the effects of a new diet found that the weight LOSS over a one-month period by those on the diet was normally distributed with a MEAN of **9** pounds and a STANDARD DEVIATION of **3** pounds. If **200** dieters were selected at random, answer the following:

a. The PROPORTION of the dieters those lost more than 12 pounds =	0.1587

X : Weight loss in pounds → $X : N(9, 9)$ → μ = 9 pounds, σ = 3 pounds

$$P(X > 12) = P\left(\frac{X-\mu}{\sigma} > \frac{12-9}{3}\right) = P(Z > 1)$$

$$= ½ - P(0 < Z < 1) = ½ - 0.3413 = \mathbf{0.1587}$$

b. The NUMBER of dieters who gained weight =	2.6 ≈ 3

To gain weight = lose 0 pounds or less → The value of $X = 0$

$$P(X < 0) = P\left(Z > \frac{0-9}{3}\right) = P(Z < -3) = P(Z > 3)$$

$$= ½ - P(0 < Z < 3) = ½ - 0.4987 = 0.0013$$

Number of dieters = 200 × 0.0013 = **2.6 ≈ 3** dieters

c. The PERCENTAGE of dieters whose their AVERAGE weight loss was between 8.5 & 9.5 pounds =	98.18%

If $X : N(9, 9)$ → $\bar{X} : N(9, 9/200)$ →

$$P\left(8.5 < \bar{X} < 9.5\right) = P\left(\frac{8.5-9}{\frac{3}{\sqrt{200}}} < Z < \frac{9.5-9}{\frac{3}{\sqrt{200}}}\right) = P(-2.36 < Z < 2.36)$$

$$= 2 \times P(0 < Z < 2.36) = 2 \times 0.4909 = 0.9818$$

→ Percentage = **98.18%**

d. If the AVERAGE weight loss of the 200 dieters was 8 pounds, do you think that the new diet regime is effective? Why?	NO

$$P\left(\bar{X} \le 8\right) = P\left(Z \le \frac{8-9}{\frac{3}{\sqrt{200}}}\right) = P\left(Z \le -\frac{\sqrt{200}}{3}\right) = P(Z \le -4.71) \approx 0$$

This means that losing on the average 8 pounds for 200 persons is VERY much LESS than the average of 9 pounds. So, the diet regime is NOT effective.

With My Best Wishes