

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS

STAT 211: BUSINESS STATISTICS I

Semester 063

Major Exam III

Monday 13/8/2007

4:30 pm - 5:30 pm

Please **circle** your section number: **2** (9:20 - 10:20 am) **3** (10:30 - 11:30 am)

Surname: **SOLUTION KEY** ID#: Serial#:

Question No	Full Marks	Marks Obtained
1	22	
2	18	
3	20	
Total	60	

Q1. (8+3+3+8 Points)

An official from the securities commission estimates that 60% of all investment bankers have profited from the use of insider information. If 10 investment bankers are selected at random from the commission's registry, answer the following:

a. The EXACT probability that at least 2 of them have profited from insider information =	0.9983
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X : # of bankers profited from insider information in a sample of 10.

$$X : B(10, 0.6) \rightarrow f(x) = \binom{10}{x} 0.6^x 0.4^{10-x}, \quad x = 0, 1, \dots, 10$$

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - \left[\binom{10}{0} 0.6^0 0.4^{10} + \binom{10}{1} 0.6^1 0.4^9 \right]$$

$$= 1 - (0.0001 + 0.0016)$$

$$= 1 - 0.0017 = \mathbf{0.9983}$$

b. The EXACT probability that all of the 10 investment bankers have profited from insider information =	0.006
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$$P(X = 10) = \binom{10}{10} 0.6^{10} 0.4^0 = \mathbf{0.006}$$

c. The AVERAGE number of investment bankers that have profited from the use of insider information =	6
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$$E(X) = np = 10(0.6) = 6 \text{ bankers}$$

d. If 100 investment bankers were selected randomly from the commission's registry, then the APPROXIMATE probability that at most half of them have profited from insider information =	0.0207
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$$\text{If } n = 100 \rightarrow np = 100(0.6) = 60 \geq 5 \text{ and } nq = 100(0.4) = 40 \geq 5$$

$$\rightarrow X : B(100, 0.6) \text{ but } X \sim N(np, npq)$$

$$\rightarrow \bar{p} \sim N(p, pq/n) \text{ but } \bar{p} \sim N(0.6, 0.24/100)$$

$$\rightarrow P(X \leq 50) = P\left(\frac{X}{n} \leq \frac{50}{100}\right) = P\left(\bar{P} \leq \frac{1}{2}\right)$$

$$= P\left(\frac{\bar{P} - p}{\sqrt{\frac{pq}{n}}} \leq \frac{\frac{1}{2} - 0.6}{\sqrt{\frac{0.24}{100}}}\right) = P\left(Z \leq \frac{-0.1}{\sqrt{0.24}}\right)$$

$$= P(Z < -2.04) = P(Z > 2.04) = \frac{1}{2} - P(0 < Z < 2.04)$$

$$= \frac{1}{2} - 0.4793 = 0.0207$$

Q2. (6+2+6+4 Points)

An advertising executive receives an average of 10 telephone calls each afternoon between 2 and 4 P.M. The calls occur randomly and independently of one another. Find the following:

a. The PROBABILITY that the executive will receive 7 calls BETWEEN 2 AND 3 P.M. on a particular afternoon =	0.1044
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X : # of calls in 2 hrs $\rightarrow \lambda = 10$ per 2 hrs = 5 per hr = 1/12 per min.

$$X: P_o(\lambda t) \rightarrow P(X = x) = f(x) = \frac{e^{-\lambda t} \lambda t^x}{x!}, \quad x = 0, 1, 2, \dots$$

When $t = 1/2 \rightarrow \lambda t = (10)(1/2) = 5$ per hr and

$$P(X = 7) = f(7) = \frac{e^{-5} 5^7}{7!} = \mathbf{0.1044}$$

b. The VARIANCE of the number of calls that the executive will receive BETWEEN 2 AND 5 P.M. on a particular afternoon =	15
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$$V(X) = \lambda t = 10 \times \frac{3}{2} = \mathbf{15} \text{ calls}$$

c. The PROBABILITY that the executive has to wait AT LEAST 15 MINUTES to receive the next call on a particular afternoon =	0.2865
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Y : Waiting time in hrs $\rightarrow Y: \text{Exp}(5) \rightarrow f(y) = 5e^{-5y}, y \geq 0$

$$P(Y \leq a) = 1 - e^{-\lambda a} \text{ and } 15 \text{ min} = 1/4 \text{ hr}$$

$$P(Y > 1/4) = 1 - P(Y < 1/4) = 1 - (1 - e^{-5/4}) = e^{-5/4} = \mathbf{0.2865}$$

d. The EXPECTED time that the executive has to wait until he receives the next TWO calls on a particular afternoon =	24 min 2/5 hr
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For ONE call only $E(Y) = 1/\lambda = 1/5$ hr

Hence for TWO calls $E(Y) = 2(1/\lambda) = (2/5) \text{ hr} = \mathbf{24} \text{ min}$

Q3. (3+5+7+5 Points)

Researchers studying the effects of a new diet found that the weight LOSS over a one-month period by those on the diet was normally distributed with a MEAN of 9 pounds and a STANDARD DEVIATION of 3 pounds. If 200 dieters were selected at random, answer the following:

a. The PROPORTION of the dieters those lost more than 12 pounds =	0.1587
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X : Weight loss in pounds $\rightarrow X : N(9, 9) \rightarrow \mu = 9$ pounds, $\sigma = 3$ pounds

$$P(X > 12) = P\left(\frac{X - \mu}{\sigma} > \frac{12 - 9}{3}\right) = P(Z > 1)$$

$$= \frac{1}{2} - P(0 < Z < 1) = \frac{1}{2} - 0.3413 = \mathbf{0.1587}$$

b. The NUMBER of dieters who gained weight =	0.26 \approx 0
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To gain weight = lose 0 pounds or less \rightarrow The value of $X = 0$

$$P(X < 0) = P\left(Z > \frac{0 - 9}{3}\right) = P(Z < -3) = P(Z > 3)$$

$$= \frac{1}{2} - P(0 < Z < 3) = \frac{1}{2} - 0.4987 = 0.0013$$

Number of dieters = $200 \times 0.0013 = \mathbf{0.26 \approx 0}$ dieters

c. The PERCENTAGE of dieters whose their AVERAGE weight loss was between 8.5 & 9.5 pounds =	98.18%
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If $X : N(9, 9) \rightarrow \bar{X} : N(9, 9/200) \rightarrow$

$$P(8.5 < \bar{X} < 9.5) = P\left(\frac{8.5 - 9}{\frac{3}{\sqrt{200}}} < Z < \frac{9.5 - 9}{\frac{3}{\sqrt{200}}}\right) = P(-2.36 < Z < 2.36)$$

$$= 2 \times P(0 < Z < 2.36) = 2 \times 0.4909 = 0.9818$$

\rightarrow Percentage = **98.18%**

d. If the AVERAGE weight loss of the 200 dieters was 8 pounds, do you think that the new diet regime is effective? Why?	NO
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$$P(\bar{X} \leq 8) = P\left(Z \leq \frac{8-9}{\frac{3}{\sqrt{200}}}\right) = P\left(Z \leq -\frac{\sqrt{200}}{3}\right) = P(Z \leq -4.71) \approx 0$$

This means that losing on the average 8 pounds for 200 persons is

VERY much LESS than the average of 9 pounds. So, the diet regime is

NOT effective.

With My Best Wishes