KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
DHAHRAN, SAUDI ARABIA

STAT 319: PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS

Major Exam III, Summer Semester (053)
Time: 5:00 – 6:15 pm, Sat. 5th July, 2006

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Student Surname: Answer Key (B) ID# Sec#: Ser.#:

Question	Full Points	Points Obtained
1	4	
2	4	
3	12	
4	8	
5	12	
6	8	
Total	48	

Q1. (4 points)

In a health study, it is given that *62%* of all the injuries occur at home. Out of a sample of *500* randomly selected injuries, *approximate* the probability that the number of home injuries will be *at most* 300 injuries.

$p = 0.62$, $n = 500$, X: # of home injuries.

$X : B(500, 0.62)$, but since ;

$np = 310 \geq 5$ & $nq = 190 \geq 5$ (1) $\Rightarrow$

$X \sim N(np, npq)$ where $np = 310$ (1) &

$\sqrt{npq} = \sqrt{500(0.62)(0.38)} = \sqrt{117.8} = 10.856$ (1),

$P(X \leq 300) \simeq P(X \leq 300.5) = P\left(Z \leq \frac{300.5 - 310}{10.856}\right)$

$\simeq P(Z \leq -0.86) = \boxed{0.1949}$ (1)

Q2. (4 points)

In a particular brick manufacturing process, typically some of the bricks produced is not suitable for all purposes. Management monitors this process by periodically collecting a random sample and classifying the bricks as suitable or unsuitable. A recent ***sample of 264*** bricks yielded ***21 unsuitable*** bricks. Construct a ***96%*** C.I. for the *true proportion* of unsuitable bricks.

Let X: # of unsuitable bricks. $\Rightarrow X : B(264, \frac{21}{264})$

But since $np = 21 \geq 5$ & $nq = 243 \geq 5 \Rightarrow$

$X \sim N(np, npq)$ & $\hat{p} \sim N(p, \frac{pq}{n})$ where

$p = \frac{21}{264} = 0.079$ (1), $\sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.079)(0.921)}{264}} = 0.0167$ (1)

To find a 96% C.I. $\Rightarrow 1-\alpha = 0.96$, $\frac{\alpha}{2} = 0.02$

$\Rightarrow Z_{\frac{\alpha}{2}} = Z_{0.02} = 2.05$ (1)

A $(1-\alpha)100\%$ C.I. for P is $\left[p \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{pq}{n}}\right] \Rightarrow$

A 96% C.I. for $P = [0.079 \pm (2.05)(0.0167)]$

$= [0.079 \pm 0.034]$

$= [0.045, 0.113]$ (1)

Q3. (4+4+4 points)

A sample of size **18** is selected from a ***normal*** population. The sample mean and the standard deviation are ***16.8 and 2.4 respectively***.

a. Construct a ***95%*** C.I. for the population *mean*.

$n = 18,\ \bar{x} = 16.8,\ S = 2.4$

To find a 95% C.I. $\Rightarrow 1-\alpha = 0.95 \Rightarrow \frac{\alpha}{2} = 0.025 \Rightarrow$

$t_{\frac{\alpha}{2}, n-1} = t_{0.025, 17} = 2.11$ ①

A $(1-\alpha)100\%$ C.I. for μ is $\left[\bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}\right]$ ① $\Rightarrow$

A 95% C.I. for $\mu = \left[16.8 \pm (2.11)\frac{2.4}{\sqrt{18}}\right]$ ①

$= [16.8 \pm 1.19] = [15.61, 17.99]$ ①

b. Construct a ***90%*** C.I. for the population *standard deviation*.

A $(1-\alpha)100\%$ C.I. for $\sigma = \left[\sqrt{\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}, n-1}}}, \sqrt{\frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}}\right]$ ①

where $1-\alpha = 0.9$ & $\frac{\alpha}{2} = 0.05 \Rightarrow$

$\chi^2_{0.05, 17} = 27.587$ ①, $\chi^2_{0.95, 17} = 8.672$ ①, $S^2 = (2.4)^2 = 5.76$

$\Rightarrow$ A 90% C.I. for $\sigma = \left[\sqrt{\frac{17(5.76)}{27.587}}, \sqrt{\frac{17(5.76)}{8.672}}\right]$

$= \left[\sqrt{\frac{97.92}{27.587}}, \sqrt{\frac{97.92}{8.672}}\right] = [1.884, 3.360]$ ①

c. Determine the required *sample size needed* for constructing a ***90%*** C.I. for μ with a *precision* of ***0.4***.

$n \geq \frac{z^2_{\frac{\alpha}{2}} S^2}{e^2}$ ① $= \frac{z^2_{0.05}(5.76)}{0.16}$ ①

$= \frac{(1.645)(5.76)}{0.16}$

$= 59.22 \approx \boxed{60}$ ①

①

a. *Q4.* (4+4 points)

According to a biological study, brain weights of Swedish men are normally distributed with a *mean* of 1.6 kg (kilograms) and a *standard deviation* of 0.14 kg. Find the following:

a. The *percentage* of Swedish men with a brain weight of *more than* 1.8 kg.

X: Brain weight of Swedish men.

$X \sim N(1.6, (0.14)^2) \Rightarrow$

① $P(X > 1.8) = P\left(Z > \frac{1.8 - 1.6}{0.14}\right)$ ①

$= P(Z > 1.43)$

$= P(Z < -1.43) = \boxed{0.0764}$ ①

Percentage $\approx \boxed{7.64\%}$ ①

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b. The weight that 43% of the weights are greater than it.

Let $p = P_{57} \Rightarrow$

$P(X < p) = 0.57$ ①

$= P\left(Z < \frac{p - 1.6}{0.14}\right)$ ①

①

$\Rightarrow \frac{p - 1.6}{0.14} = 0.18 \Rightarrow p = 1.6252$ kg

$\Rightarrow \boxed{P_{57} = 1.6252 \text{ kg}}$ ①

Q5. (4+4+4 points)

The lifetimes of electrical appliances produced from an assembly line have the ***exponential*** distribution with a mean of 12 years.

a. What is the probability that a randomly selected appliance will survive ***at least*** for 9 years?

Let X : Lifetime of electrical appliances.

(1) $X : Exp(12) \Rightarrow f(x) = \begin{cases} \frac{e^{-x/12}}{12}, & x \geq 0 \\ 0, & e.w. \end{cases}$ (1)

(1) $P(X \geq 9) = \int_9^\infty f(x)\,dx = \frac{1}{12}\int_9^\infty e^{-x/12}\,dx = -e^{-x/12}\Big]_9^\infty$

$= -0 + e^{-9/12} = e^{-3/4} = \boxed{0.472}$ (1)

b. Find the ***median*** of the lifetime of the electrical appliance.

Let Median = m $\Rightarrow F(m) = \frac{1}{2} = P(X \leq m)$ (1)

$\Rightarrow F(m) = \int_0^m f(x)\,dx = \frac{1}{2} = \int_0^m \frac{e^{-x/12}}{12}\,dx$ (1)

$\Rightarrow -e^{-x/12}\Big]_0^m = \frac{1}{2} \Rightarrow 1 - e^{-m/12} = \frac{1}{2}$ (1) $\Rightarrow$

$e^{-m/12} = \frac{1}{2} \Rightarrow m = -12 \ln \frac{1}{2} = \boxed{8.32}$ years (1)

c. If a sample of size 50 appliances was selected, what is the probability that the ***average*** lifetimes of the 50 appliances will exceed 12.5 years?

$n = 50 \geq 30 \Rightarrow$ by (1) CLT $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ where

$\mu = E(X) = 12$ (1), $\sigma = 12 \Rightarrow \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{50}} = 1.7$ (1)

$P(\bar{X} > 12.5) = P\left(Z > \frac{12.5 - 12}{1.7}\right)$

$= P(Z > 0.29)$

$= P(Z < -0.29) = \boxed{0.3859}$ (1)

Q6. (4+4 points)
It is given that on a certain dangerous road, three traffic accidents take place every 15 km, then;

a. What is the probability that one needs to travel ***no more than*** 5 km before an accident occurs?

1

$\lambda = 3/15$ km, X: Distance traveled to get one acc.

$X: Exp(\frac{1}{\lambda}) = Exp(5) \Rightarrow f(x) = \begin{cases} \frac{e^{-x/5}}{5}, & x \geq 0 \\ 0, & e.w. \end{cases}$

1

$F(x) = P(X \leq x) = \int_0^x f(t)dt = \int_0^x \frac{e^{-t/5}}{5} dt$

$= -e^{-t/5}\Big]_0^x = \begin{cases} 1 - e^{-x/5}, & x \geq 0 \\ 0, & e.w. \end{cases}$

$P(X < 5) = F(5) = 1 - e^{-5/5} = 1 - e^{-1} = 1 - \frac{1}{e}$ 1

$= 1 - 0.368 = \boxed{0.632}$ 1

b. What is the ***expected*** distance to be traveled to encounter **4** accidents?

1

Let X: Distance traveled to encounter 4 accidents.

1

$\Rightarrow X : \Gamma(4, 5), \; \alpha = 4, \; \beta = \frac{1}{\lambda} = 5$

$\Rightarrow E(X) = \alpha\beta = 4 \times 5 = \boxed{20}$ km

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