KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
DHAHRAN, SAUDI ARABIA

STAT 319: PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS

Major Exam III, Summer Semester (053)
Time: 5:00 – 6:15 pm, Sat. 5^{th} July, 2006

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Student Surname: Answer Key (A) ID# Sec#: Ser.#:

Question	Full Points	Points Obtained
1	4	
2	4	
3	12	
4	8	
5	12	
6	8	
Total	48	

Q1. (4 points)

In a particular brick manufacturing process, typically some of the bricks produced is not suitable for all purposes. Management monitors this process by periodically collecting a random sample and classifying the bricks as suitable or unsuitable. A recent ***sample of 214*** bricks yielded ***18 unsuitable*** bricks. Construct a ***98%*** C.I. for the *true proportion* of unsuitable bricks.

Let X: # of unsuitable bricks (1)

$\Rightarrow X : B(214, \frac{18}{214})$, $p = \frac{18}{214} = 0.084$ but

Since $np = 18 \geq 5$ & $nq = 196 \geq 5 \Rightarrow$

$X \sim N(np, npq)$ & $\hat{p} \sim N(p, \frac{pq}{n})$ (1)

$\Rightarrow \hat{p} \sim N(0.084, 0.00036) \Rightarrow \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = 0.0189$

To find a 98% CI. $\Rightarrow 1-\alpha = 0.98$ & $\frac{\alpha}{2} = 0.01 \Rightarrow$

$z_{\frac{\alpha}{2}} = z_{0.01} = 2.33$ (1),

A $(1-\alpha)100\%$ C.I. for P is $[p \pm z_{\frac{\alpha}{2}}\sqrt{\frac{pq}{n}}] \Rightarrow$

A 98% CI. for $P = [0.084 \pm (2.33)(0.0189)] = [0.084 \pm 0.044]$

$= [0.04, 0.128]$ (1)

Q2. (4 points)

In a health study, it is given that 73% of all the injuries occur at home. Out of a sample of *600* randomly selected injuries, *approximate* the probability that the number of home injuries will be *at most* 360 injuries.

$p = 0.73$, $n = 600$, X: # of home injuries.

$X : B(600, 0.73)$ but since; (1)

$np = 600(0.73) = 438 \geq 5$ & $nq = 162 \geq 5 \Rightarrow$ (1)

$X \sim N(np, npq)$ where $np = 438$ & (1)

$\sqrt{npq} = \sqrt{600(0.73)(0.27)} = 10.875$, Then

$P(X \leq 360) \simeq P(X \leq 360.5)$

$\simeq P(Z \leq \frac{360.5 - 438}{10.875})$

$\simeq P(Z \leq -7.13) = \boxed{0}$ (1)

Q3. (4+4+4 points)

The lifetimes of electrical appliances produced from an assembly line have the ***exponential*** distribution with a <u>*mean of 10*</u> years.

a. What is the probability that a randomly selected appliance will survive ***at least*** for 15 years?

Let X: Liftime of electrical appliances.

(1) $X: \text{Exp}(10) \Rightarrow f(x) = \begin{cases} \frac{e^{-x/10}}{10}, & x \geq 0 \\ 0, & \text{o.w.} \end{cases}$ (1)

(1)

$P(X \geq 15) = \int_{15}^{\infty} f(x)\,dx = \frac{1}{10}\int_{15}^{\infty} e^{-x/10}\,dx = -e^{-x/10}\Big]_{15}^{\infty}$

$= -0 + e^{-15/10} = e^{-3/2} = \boxed{0.223}$ (1)

b. Find the ***median*** of the lifetime of the electrical appliance.

Let the median be $m \Rightarrow F(m) = \frac{1}{2} = P(X \leq m)$ (1)

$\Rightarrow F(m) = \int_0^m f(x)\,dx = \frac{1}{2} = \int_0^m \frac{1}{10} e^{-x/10}\,dx$ (1)

$\Rightarrow -e^{-x/10}\Big]_0^m = \frac{1}{2} \Rightarrow 1 - e^{-m/10} = \frac{1}{2}$ (1) $\Rightarrow$

$e^{-m/10} = \frac{1}{2} \Rightarrow m = -10 \ln \frac{1}{2} = \boxed{6.93}$ years (1)

c. If a sample of size 100 appliances was selected, what is the probability that the ***average*** lifetimes of the 100 appliances will <u>*exceed 10.5*</u> years?

$n = 100 \geq 30 \Rightarrow$ by (1) CLT $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ where

$\mu = E(X) = 10$ (1) $, \sigma = 10 \Rightarrow \frac{\sigma}{\sqrt{n}} = \frac{10}{10} = 1$ (1)

$P(\bar{X} > 10.5) = P(Z > \frac{10.5 - 10}{1})$

$= P(Z > \frac{1}{2})$

$= P(Z < -\frac{1}{2}) = \boxed{0.3085}$ (1)

Q4. (4+4 points)

According to a biological study, brain weights of Swedish men are normally distributed with a *mean* of 1.4 kg (kilograms) and a *standard deviation* of 0.11 kg. Find the following:

a. The *percentage* of Swedish men with a brain weight of *more than* 1.6 kg.

X: Brain weight of Swedish men.

① $P(X > 1.6) = P\left(Z > \frac{1.6 - 1.4}{0.11}\right)$ ①

$= P(Z > 1.82)$

$= P(Z < -1.82) = \boxed{0.0344}$ ①

Percentage $\approx \boxed{3.44\%}$ ①

b. The weight that 32% of the weights are greater than it.

Let $p = P_{68} \Rightarrow$

$P(X < p) = 0.68$ ①

$= P\left(Z < \frac{p - 1.4}{0.11}\right)$ ①

① $\Rightarrow \frac{p - 1.4}{0.11} = 0.47 \Rightarrow p = 1.4517$ kg

$\Rightarrow \boxed{P_{68} = 1.4517 \text{ kg}}$ ①

Q5. (4+4+4 points)

A sample of size *20* is selected from a ***normal*** population. The sample mean and the standard deviation are ***18.6 and 4.2 respectively***.

a. Construct a ***98%*** C.I. for the population mean.

$n = 20$, $\bar{x} = 18.6$, $s = 4.2$,

To find a 98% C.I. $\Rightarrow 1-\alpha = 0.98 \Rightarrow \frac{\alpha}{2} = 0.01$

$\Rightarrow t_{\frac{\alpha}{2}, n-1} = t_{0.01, 19} = 2.539$ (1)

A $(1-\alpha)100\%$ C.I. for μ is $\left[\bar{X} \pm t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}\right]$ (1)

$\Rightarrow$ A 98% C.I. for $\mu = \left[18.6 \pm (2.539) \frac{4.2}{\sqrt{20}}\right]$ (1)

$= [18.6 \pm 2.38] = \boxed{[16.22, 20.98]}$ (1)

b. Construct a ***90%*** C.I. for the population standard deviation.

A $(1-\alpha)100\%$ for $\sigma = \left[\sqrt{\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}, n-1}}}, \sqrt{\frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}}\right]$ (1)

where $1-\alpha = 0.9$ & $\frac{\alpha}{2} = 0.05 \Rightarrow$

$\chi^2_{0.05, 19} = 30.144$ (1), $\chi^2_{0.95, 19} = 10.117$ (1), $S^2 = (4.2)^2 = 17.64$

$\Rightarrow$ A 90% C.I. for $\sigma = \left[\sqrt{\frac{19(17.64)}{30.144}}, \sqrt{\frac{19(17.64)}{10.117}}\right]$

$= \left[\sqrt{\frac{335.16}{30.144}}, \sqrt{\frac{335.16}{10.117}}\right] = \boxed{[3.334, 5.756]}$ (1)

c. Determine the required sample size needed for constructing a ***90%*** C.I. for μ with a precision of ***0.5***.

$n \geq \frac{z^2_{\frac{\alpha}{2}} s^2}{e^2}$ (1) $= \frac{z^2_{0.05}(17.64)}{0.25}$ (1)

$= \frac{(1.645)^2(17.64)}{0.25}$

$= 190.94$ (1) $\approx \boxed{191}$ (1)

Q6. (4+4 points)

It is given that on a certain dangerous road, two traffic accidents take place every 20 km, then;

a. What is the probability that one needs to travel ***no more than*** 10 km before an accident occurs?

1

$\lambda = 2/20\text{km}$, X: Distance traveled to get one acc.

$X: Exp(\frac{1}{\lambda}) = Exp(10) \Rightarrow f(x) = \begin{cases} \frac{e^{-x/10}}{10}, & x \geq 0 \\ 0, & e.w. \end{cases}$ 1

$F(x) = P(X \leq x) = \int_0^x f(t)\,dt = \int_0^x \frac{e^{-t/10}}{10}\,dt$

$= -e^{-t/10}\Big]_0^x = \begin{cases} 1 - e^{-x/10}, & x \geq 0 \\ 0, & e.w. \end{cases}$

$P(X < 10) = F(10) = 1 - e^{-10/10} = 1 - e^{-1} = 1 - \frac{1}{e}$ 1

$= 1 - 0.368 = \boxed{0.632}$ 1

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b. What is the ***expected*** distance to be traveled to encounter ***5*** accidents?

1

Let X: Distance traveled to encounter 5 accidents.

1 $X: \Gamma(5, 10)$, $\alpha = 5$, $\beta = \frac{1}{\lambda} = 10$

$\Rightarrow E(X) = \alpha\beta = 5 \times 10 = \boxed{50}$ km

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