CHAPTER FOUR
Continuous Random Variables

The following table contains some of the most well known, and often used continuous distributions in Engineering.

Table 1 Some Continuous Random Variables and Their Means and Variances

Distribution	Density Function	Mean	Variance
Exponential	$f(x) = \lambda\, e^{-\lambda x}$, $x > 0,\ \lambda > 0$	$1/\lambda$	$1/\lambda^2$
Normal	$f(x) = \left(2\pi\sigma^2\right)^{-1/2} \quad e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, $-\infty < x < \infty$	μ	σ^2
Gamma	$f(x) = \dfrac{x^{\alpha-1}e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)}$, $0 \le x < \infty,\ \ 0 < \alpha < \infty,\ \ 0 < \beta < \infty,$	$\alpha\beta$	$\alpha\beta^2$
Weibull	$f(x) = \dfrac{\alpha}{\beta}\ x^{\alpha-1}e^{-x/\beta}$, $0 \le x < \infty,\ \ 0 < \alpha < \infty,\ \ 0 < \beta < \infty.$	$\beta^{1/\alpha}\Gamma(1+1/\alpha)$	$\beta^{2/\alpha}\left[\Gamma(1+2/\alpha) - \Gamma^2(1+1/\alpha)\right]$
Lognormal	$f(x) = \dfrac{1}{x\sigma\sqrt{2\pi}} e^{-(\ln x-\mu)^2/2\sigma^2}$, $0 < x < \infty,\ \ \mu > 0,\ \ \sigma > 0$	μ	σ^2
Beta	$f(x) = \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\ x^{\alpha-1}(1-x)^{\beta-1}$, $0 \le x \le 1,\ \ 0 < \alpha < \infty,\ \ 0 < \beta < \infty$	$\dfrac{\alpha}{\alpha+\beta}$	$\dfrac{\alpha}{\alpha+\beta}\ \dfrac{\beta}{(\alpha+\beta)(\alpha+\beta+1)}$

4.1 The Probability Calculator

The probability calculator can be used to compute probabilities for continuous random variables. It is accessed through the ***Basic Statistics and tables*** module following the steps:

(1) Statistics/ Basic Statistics and Tables (getting Figure 4.1)
(2) Probability Calculator
(3) OK.

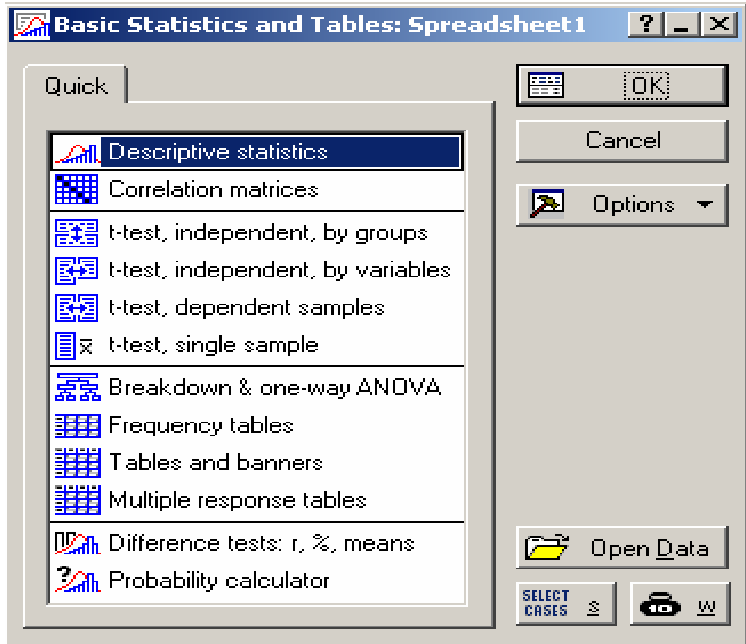

Figure 4.1 Basic Statistics and Tables: Spreadsheet

By default, this opens the probability distribution calculator menu for the Beta random variable with shape parameters 2 and 2 as shown in Figure 4.2.

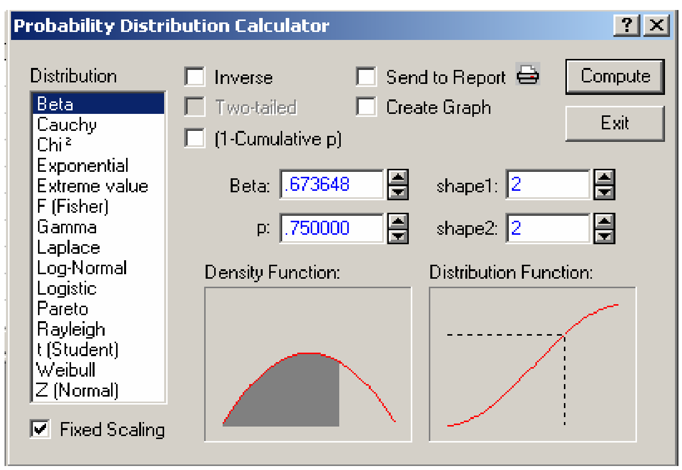

Figure 4.2 Probability Distribution Calculator

To compute the probability for a particular continuous random variable, its distribution is highlighted and the appropriate parameters are supplied. It is helpful to view the graph of the density function as it gives visual insight of the type of probability function under consideration. Note that for continuous random variables, the probability is represented by the area under the probability density function. In case the density function is invisible, uncheck ***Fixed Scaling*** (see the bottom left corner of Figure 4.2).

The ***Inverse, Two-tailed*** and **(*1-Cumulative p*)** functions are available for calculation of the probability of an event or calculation of a quartile. While using the probability calculator, it is important to view the shaded part of the graph of the density function and make sure that the shaded part corresponds to the event of interest.

4.2 The Exponential Distribution

Example 4.1 Life length of a particular type of battery follows exponential distribution with mean 2 hundred hours. Find the probability that the

(a) life length of a particular battery of this type is less than 2 hundred hours.
(b) life length of a particular battery of this type is more than 4 hundred hours.
(c) life length of a particular battery of this type is less than 2 hundred hours or more than 4 hundred hours.

Solution Let X = life length of a battery. Then $X \sim Exp(1/2)$, by $\mu = 1/\lambda = 2$ (given) so that $\lambda = 1/2 = 0.5$

(a) $P(X<2) = F(2) = 1 - e^{-(0.5)2} = 1 - e^{-1} = 0.632$

(b) $P(X>4) = 1 - P(X \le 4) = 1 - \left[1 - e^{0.5(4)}\right] = e^{-2} = 0.135$

(c) $P\left[(X<2) \text{ or } (X>4)\right] = P(X<2) + P(X>4) = 1 - e^{-1} + e^{-2} = 0.767$

Computing Exponential Probabilities Using Statistica
To compute the probability of an event related to the exponential random variable, select ***Exponential*** from the distribution list of the ***Probability Distribution Calculator*** (see Figure 4.3), next supply ***lambda***, and the value of *x* in the exp. slot.

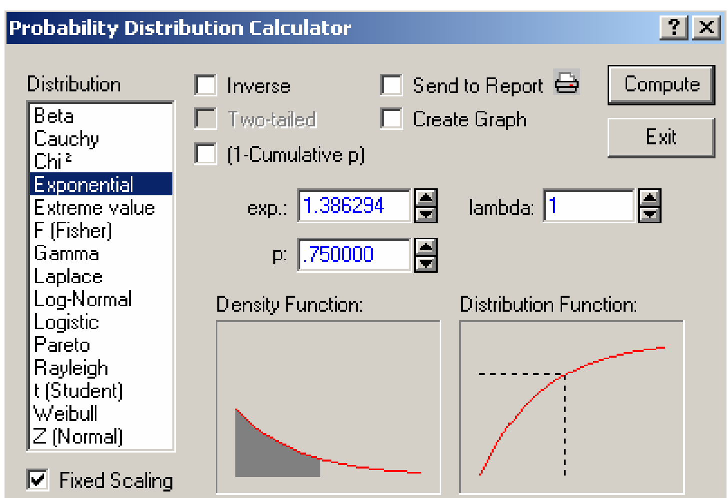

Figure 4.3 Exponential probability calculator

In the *Exponential probability calculator*, we have the following two cases:

Case 1: Leaving the ***Inverse*** and ***(1- cumulative p)*** unchecked

If the ***Inverse*** and ***(1-cumulative p)*** functions are unchecked, a figure similar to Figure 4.3 is obtained for the exponential random variable. The shaded area under the density function indicates a probability of the form $P(X < u) = p$ where $u = 1.386294$ is the value of X, $\lambda = 1$ and $p = 0.75$. That is, Figure 4.3 shows that $P(X < 1.386294) = 0.75$ where X is an exponential random variable with parameter $\lambda = 1$.

So to compute the probability that the exponential random variable is less than or equal to 2 in part (a) in Example 4.1, enter 2 for the value of x in the exp. slot and enter 0.5 for *lambda* and then click ***Compute*** to read the probability value in the slot for p.

To find u such that $P(X < u) = 0.82$ where the random variable X has an exponential distribution with parameter $\lambda = 3$, put 0.82 for p, 3 for *lambda*, and click compute to get u = 0.571599 which is in the exp. slot. Gradually we will be using the notation u = 0.571599 = $x_{0.18}$ meaning that $P(X > x_{0.18}) = 0.18$ and $P(X \le x_{0.18}) = 0.82$.

Case 2: (*1-Cumulative p*) checked

When ***(1-Cumulative p)*** is checked for the exponential random variable as shown in Figure 4.4, we obtain a probability of the form $P(X > u)$. To find $P(X > 4)$ where X has an exponential distribution with parameter 2, check ***(1-Cumulative p)***, put 2 for *lambda* and 4 for X in the exp. slot, then click compute, to get $p = 0.000335$, *i.e.*, $P(X > 4) = 0.000335$ (Figure 4.4).

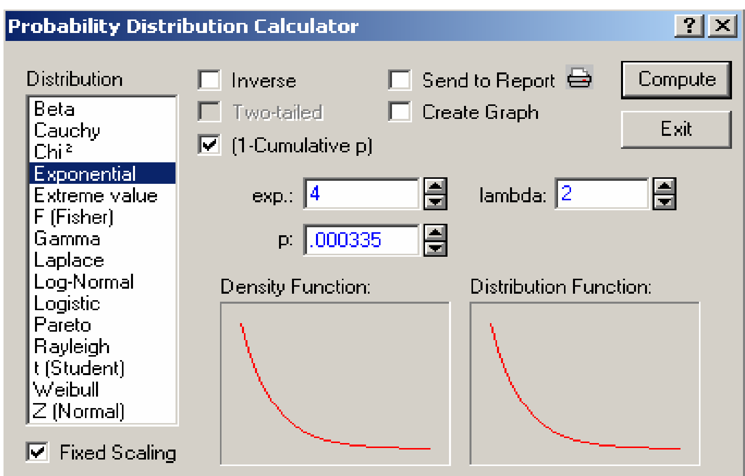

Figure 4.4 Checking (1-Cumulative p) only

It is possible to find u such that $P(X \ge u) = 0.75$ where X has an exponential distribution with parameter $\lambda = 3$. Check ***(1-Cumulative p),*** put 0.75 for p and 3 for *lambda*, click "Compute" to get the value of $u = 0.095894$ in the exp. Slot, i.e,

$P(X \geq u) = 0.75 \quad \Rightarrow u = 0.095894 = y_{0.75}$, which is the 25th percentile of exponential distribution with parameter 3.

4.3 The Normal Distribution

When the mean of a normal distribution equals 0, and the variance equals 1, we get what we call a standard normal random *Z*. Its density is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} \quad e^{-z^2/2} \, , \qquad -\infty < z < \infty$$

Computing Standard Normal Probabilities Using Tables
Using the standard normal probability table in Appendix A2 we can find the following:
$P(Z < 2.13) = 0.9834$
$P(Z > -1.68) = 0.9535$
$P(-1.02 < Z < 1.51) = 0.9345 - 0.1562 = 0.7783$

Computing Normal Probabilities Using Tables
Example 4.2 A manufacturing process has a machine that fills coke to 300 ml bottles. Over a long period of time, the average amount dispensed into the bottles is 300 ml, but there is a standard deviation of 5 ml in this measurement. If the amounts of fill per bottle can be assumed to be normally distributed, find the probability that the machine will dispense between 295 and 310 ml of liquid in any one bottle. (cf. Scheaffer and McClave, 1995, 216-217).

Solution Let X = amount of fill in a bottle. Then $X \sim N(300, \; 5^2)$.

$$\begin{aligned} P(295 < X < 310) &= P\left(\frac{295-300}{5} < \frac{X-300}{5} < \frac{310-300}{5} \right) \\ &= P(-1 < Z < 2) \\ &= 0.9772 - 0.1587 = 0.8185 \end{aligned}$$

Example 4.3 The compressive strength of samples of cement can be modeled by a normal distribution with a mean of 6000 kilograms per square centimeter and a standard deviation of 100 kilograms per square meter.

(a) What is the probability that the strength of a sample is less than 6164.5 kg/cm^2?
(b) What compressive strength is exceeded by 95% of the time?
(c) What compressive strength exceeds 5% of the time?

Solution (a) $P(X < 6164.5) = P\left(\frac{X - 6000}{100} < \frac{6164.5 - 6000}{100} \right) = P(Z < 1.645) \approx 0.95$

(b) $P(X > u) = 0.95 \Rightarrow P\left(\frac{X-\mu}{\sigma} > \frac{u-\mu}{\sigma} \right) = 0.95 \Rightarrow P\left(Z > \frac{u-\mu}{\sigma} \right) = 0.95$

From the Standard Normal Probability Table, $P(Z > -1.645) = 0.95$ so that by comparison we have $\frac{u-\mu}{\sigma} = -1.645 \Rightarrow u = \mu - 1.645\sigma = 5835.5$.

Computing Standard Normal Probabilities Using Statistica

To compute probabilities for a standard normal random variable, select ***Z(Normal)*** from the distribution list of the ***Probability Distribution Calculator*** **(**see Figure 4.5), next supply the ***mean*** and the ***standard deviation*** **(*st. dev.*).**

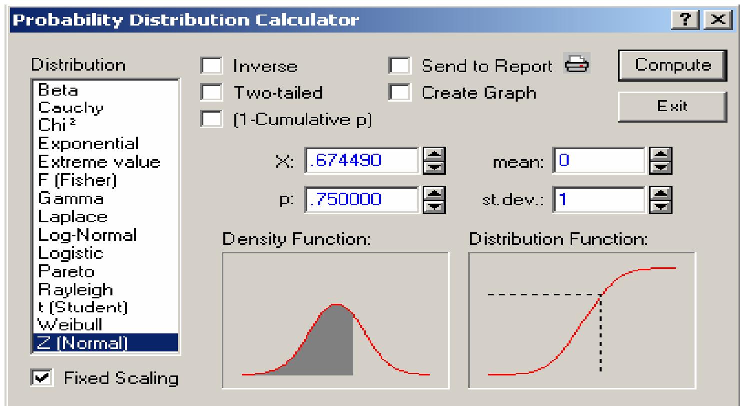

Figure 4.5 The Z (Normal) Probability Calculator

Case 1: Leaving the ***Inverse, Two-tailed*** and **(*1- Cumulative* p*)*** unchecked

The default option is $P(Z < a)$. To calculate $P(Z < 0.67449)$, put 0.67449 (=a) for x and click "compute" to get the required probability to be 0.75, that is, $P(Z < 0.67449) = 0.75$ as in Figure 4.5. The quantity x in the figure is the value of the standard normal random variable Z .

To calculate the 75th percentile i.e. to find a such that $P(Z \leq a) = 0.75$, enter 0.75 for p and click "compute" to get 0.67449 i.e. $P(Z < 0.67449) = 0.75$ meaning a = 0.67449 = $z_{0.25}$, the 75th percentile or the third quartile of the standard normal random variable.

Case 2: Only ***Two-tailed*** checked

Check only ***Two-tailed*** to calculate a probability of the form $P(-a < Z < a)$. For example, to calculate $P(-1.439531 < Z < 1.439531)$ put 1.439531 for x and click "compute" to get 0.85 under p, see Figure 4.6.

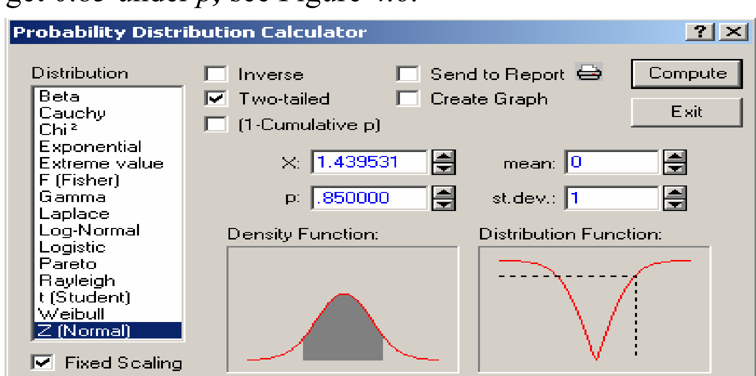

Figure 4.6 Only Two-tailed Checked

Case 3: Only ***(1-Cumulative p)*** checked

To evaluate $P\left(Z > 1.439531\right)$, click ***(1-Cumulative p),*** put 1.439531 for x and click "compute" to get 0.075, i.e. $P(Z > 1.439531) = 0.075$ (see Figure 4.7).

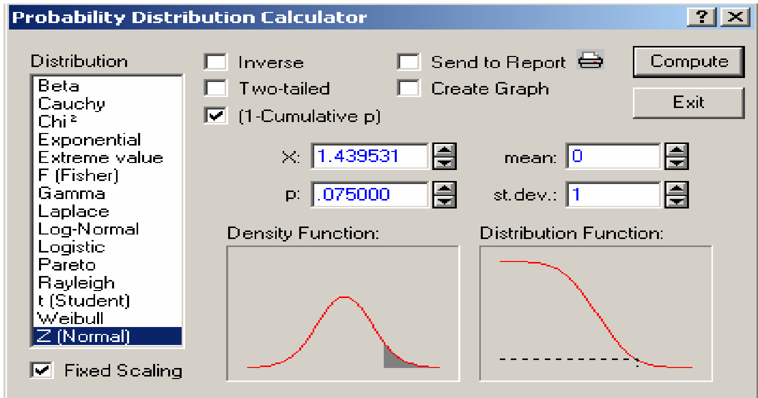

Figure 4.7 Only (1-cumulative p) Checked

Let z_α denote the $(1-\alpha)100^{th}$ percentile of Standard Normal Distribution. For example $P(Z < 1.959964) = (1 - 0.975)$ or equivalently $P(Z > 1.959964) = 0.025$.
Thus, $z_{0.025} = 1.959964 \approx 1.96$, which is the 97.5th percentile of the Standard Normal Distribution. You can check that the quartiles of the distribution are given by $z_{0.75} = -0.67449$, $z_{0.50} = 0$ and $z_{0.25} = 0.67449$.

To find the 92.5th percentile, i.e. to find a such that $P\left(Z > a\right) = 0.075$, check ***(1-Cumulative p),*** put 0.075 for p and click "compute" to get 1.439531, i.e. $P\left(Z > 1.439531\right) = 0.075$ (see Figure 4.7).

Case 4: ***Two-tailed*** and ***(1- Cumulative p)*** checked

If both the ***Two-tailed*** and ***(1-Cumulative p)*** are checked for the standard normal random variable, then the probability being computed is of the form

$$P(|Z| > a) = P(Z < -a \text{ or } Z > a) = P\left(Z < -a\right) + P\left(Z > a\right).$$

Thus, to calculate $P\left(\left|Z\right| > 1.439531\right)$, check both ***Two-tailed*** and ***(1-Cumulative p)*** function put 1.439531 for x and click "compute" to get 0.15. i.e. $P\left(|Z| > 1.439531\right)$ = $P(Z < -1.439531 \text{ or } Z > 1.439531)$ = 0.15. (See Figure 4.8).

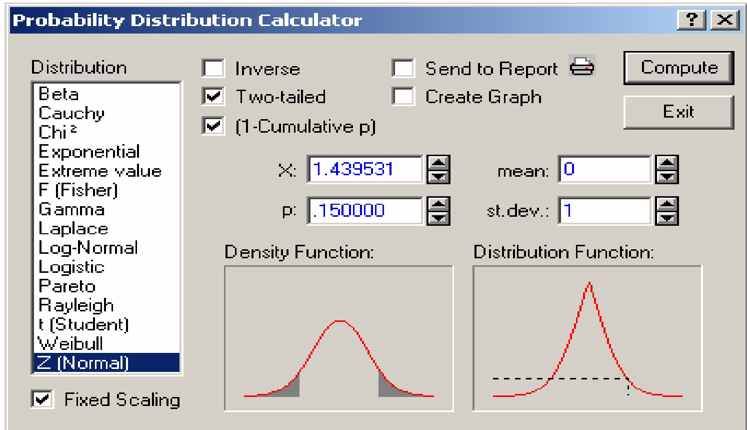

Figure 4.8 Two-tailed and (1-Cumulative p) Checked

Case 5: *Two-tailed* and ***Inverse*** checked

Similarly, one can obtain the value of a, such that $P(|Z| > a) = 0.15$ by checking both the ***Two-tailed*** and ***Inverse***, and putting 0.15 for ***p*****.** The solution to a is 1.439531 in the slot for x.

Finding the Values z_α and $z_{\alpha/2}$ of the Standard Normal Random Variable

The value of the standard random variable for which the probability is α to its right is denoted by z_α and α is called the tail probability. For instance, if we have $P(Z > 1.439531) = 0.075$, then the value 1.439531 has a probability of 0.075 to its right, i.e., $z_{0.075} = 1.439531$ (see Case 3 and Figure 4.7 above).

$z_{\alpha/2}$ is the value of the standard random variable having probability (or an area) of $\alpha/2$ to its right. Given the value of α, we may find the value of $z_{\alpha/2}$ by checking the ***Two-tailed*** and **(*1-Cumulative p*)** together, and entering α for p in the ***standard normal probability calculator***. The value computed for x is then read as the $z_{\alpha/2}$ value. For example, in Figure 4.8, $z_{\alpha/2}$ is computed in x as 1.439531, where $\alpha = 0.15$. This provides $z_{0.075} = 1.439531$.

Alternatively, the value of $z_{\alpha/2}$ may be found by first finding α/2 and using it for p as in Case 3 (see Figure 4.7)**.**

Probabilities of Normal Random Variables Using Statistica

All the illustrations have been done so far using the standard normal random variable. In the case of normal random variables, the principle remains the same, but care needs to be taken in the interpretation of the ***two-tailed*** probabilities.

Case 1: Leaving the ***Inverse, Two-tailed and 1-Cumulative p*** unchecked

To calculate $P(X < a)$ where $X \sim N(\mu, \sigma^2)$ with $\mu = 60$, $\sigma = 25$ and $a = 90$ say, put 60 for mean, 25 for standard deviation and 90 for 'x '. Beware that 'x ' in Figure 4.9 is the value 'a ' of random variable X.

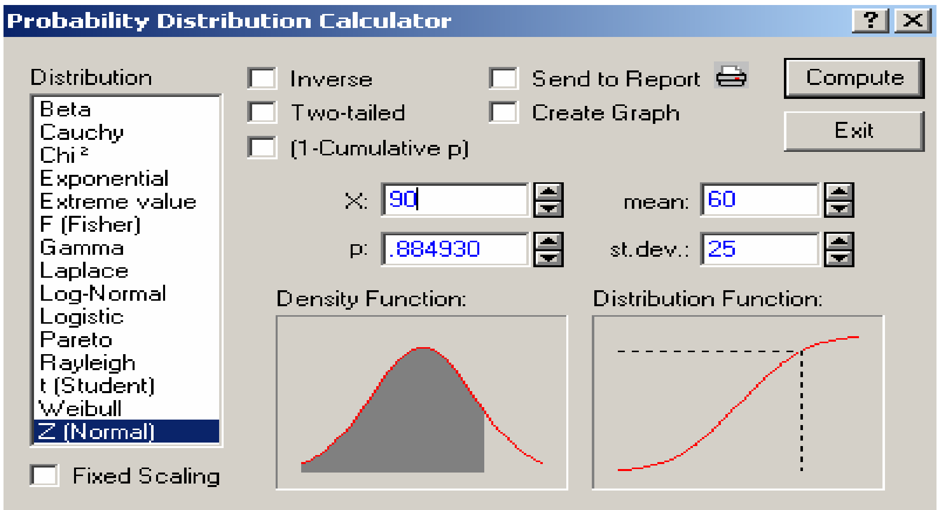

Figure 4.9 Normal Distribution with Inverse, Two-tailed and (1-Cumulative p) unchecked

Case 2: Normal Variable with only ***Two-tailed*** checked

To calculate $P(\mu - a < \mu + a)$ click *Two-tailed* and proceed, for example to calculate $P(30 < X < 90)$, where X is a normal random variable with $\mu = 60, \sigma = 25$ enter 60 for mean, 30 for standard deviation and 90 for x. This provides 0.769861 for p, i.e., $P(30 < X < 90) = 0.769861$, see Figure 4.10. Note that if the interval (a, b) is not symmetric about the mean, then compute $P(a < X < b)$ as $P(a < X < b) = P(X < b) - P(X < a)$.

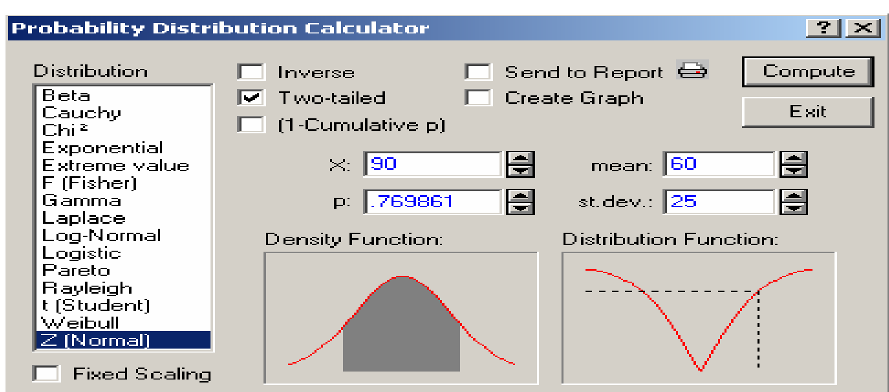

Figure 4.10 Normal Distribution with only Two-tailed Checked

Case 3: Normal Variable with ***Two-tailed*** and ***(1-Cumulative p)*** checked

Figure 4.11 shows the probability calculator for the normal variable having the mean of 60 and standard deviation of 25 with both ***Two-tailed*** and ***(1-Cumulative p)*** checked. To calculate $P(X < \mu - a \quad or \quad X > \mu + a)$, chick both ***Two-tailed*** and **(*1-Cumulative p*).** Let $a = 30$, $\mu = 60$ and $\sigma = 25$. To calculate $P(X < 30 \quad or \quad X > 90)$ = $P(X < 30) + P(X > 90)$, put 90 for *x* to get 0.230139 for *p*, i.e., $P(X < 30) + P(X > 90) = 0.230139$.

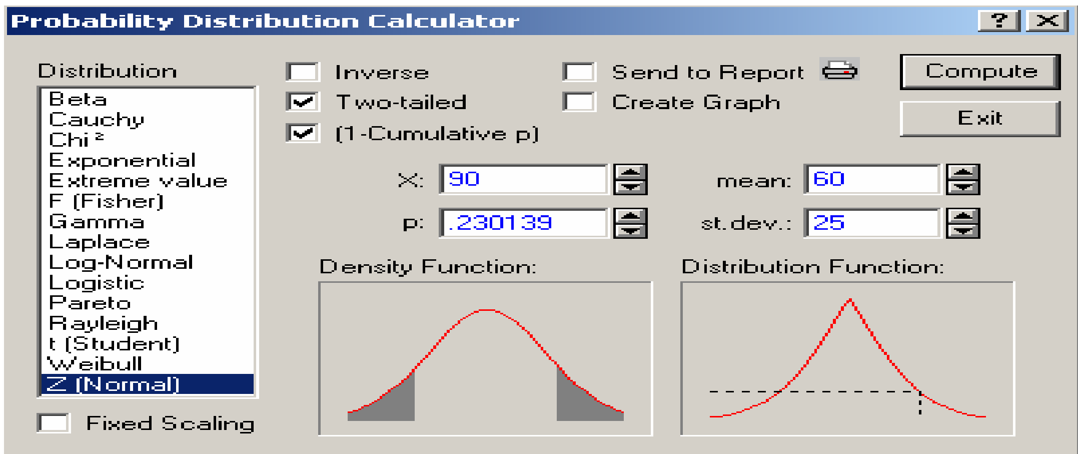

Figure 4.11 Normal distribution with Two-tailed and (1-cumulative p) Checked

4.4 Other Distributions

The Gamma Probability Calculator

The probability density function of the gamma random variable is given by

$$f(x) = \frac{x^{\alpha-1} e^{-x}}{\Gamma(\alpha)}, \quad 0 \le x < \infty, \quad 0 < \alpha < \infty$$

The ***Gamma*** probability calculator provides probability of events of the type $X < a$ for the Gamma distribution with shape parameter α.

To calculate *P*(*X*< 1.386294), where *X* has Gamma distribution with shape parameter α = 1, put 1.386294 for *G* (the value of *X*) and 1 for shape, then click “compute” to get 0.75 for ***p*,** i.e. *P*(*X*< 1.386294) = 0.75 (see Figure 4.12).

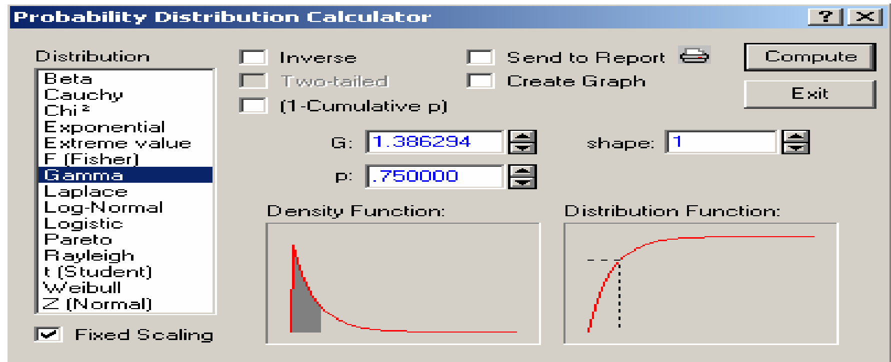

Figure 4.12 Probability Calculator for the Gamma Distribution

Similarly, to calculate *P*(*X*< 1.386294), where *X* has a Gamma distribution with α = 2, put 2 for the shape and 1.386294 for *G* and click "compute" to get 0.403426 for *p*, i.e., *P*(*X*< 1.386294) = 0.403426.

The Weibull Probability Calculator

The Weibull Probability calculator provides the probability of events of the type $W < a$ for scale parameter β and shape parameter α of Weibull distribution. Plots of the probability density function and cumulative distribution function are also available.

To calculate $P(W < 1.386294)$, put 1.386294 for *w,* 1 for shape and 1 for scale, then click "compute" to get 0.75 for p, i.e. $P(W < 1.386294) = 0.75$ (see Figure 4.13).

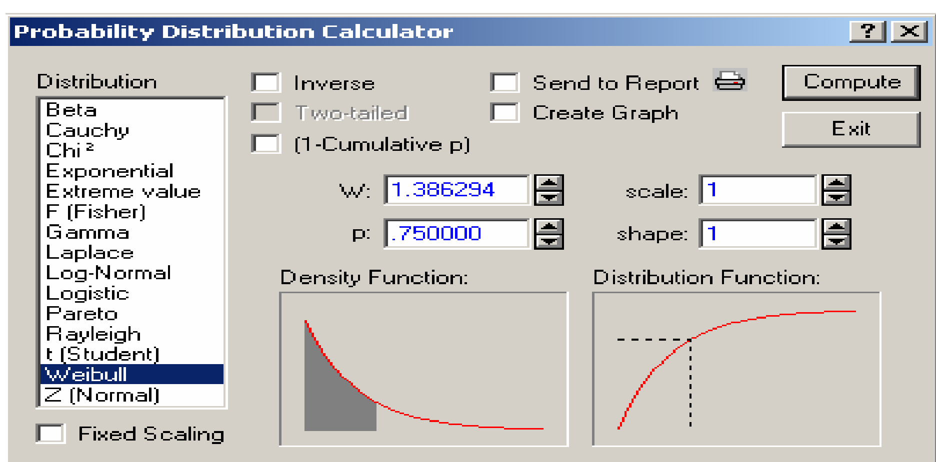

Figure 4.13 Probability Calculator for the Weibull Distribution

Similarly, for the ***Weibull*** distribution with scale parameter $\beta = 3$ and shape parameter $\alpha = 2$, one can calculate $P(W < 1.386294)$, by putting 2 for the shape, 3 for the scale and 1.386294 for *w* (which is the value of *W*) and click "compute" to get 0.192276 for *p*, i.e., $P(W < 1.386294) = 0.192276$.

The Lognormal Probability Calculator

It computes the integral and inverse integral for the Lognormal (Scale μ, Shape σ) distribution. Plots of the probability density function and cumulative distribution function are also available. To find *lognormal* probabilities using Statistica, we select **Log-Normal** from the distribution list to get Figure 4.14.

Here, we are required to supply the two parameters μ and shape parameter σ. By default Statistica gives the the lognormal distribution with μ = 0 and σ = 1. To calculate $P\left(X < 1.963031\right)$, where X has a lognormal distribution with μ = 0 and σ = 1, put 1.963031 for *L*, then click 'compute' to get 0.75 for p, i.e., $P\left(X < 1.963031\right) = 0.75$.

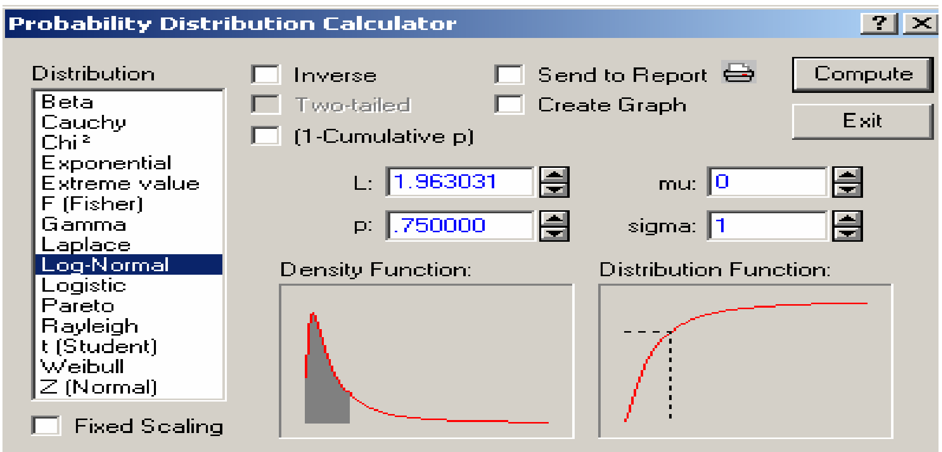

Figure 4.14 Probability Calculator for the Log-Normal Distribution

Similarly, for the ***lognormal*** distribution with scale parameter μ = 2 and shape parameter σ = 5, one can calculate $P\left(X < 1.963031\right)$, by putting 2 for **mu,** 5 for **sigma,** 1.963031 for *L*, and click "compute" to get 0.395465 for *p*, i.e., $P\left(X < 1.963031\right) = 0.395465$.

The Beta Probability Calculator

To find the beta probability using Statistica, we select **Beta** from the distribution list (See Figure 4.15).

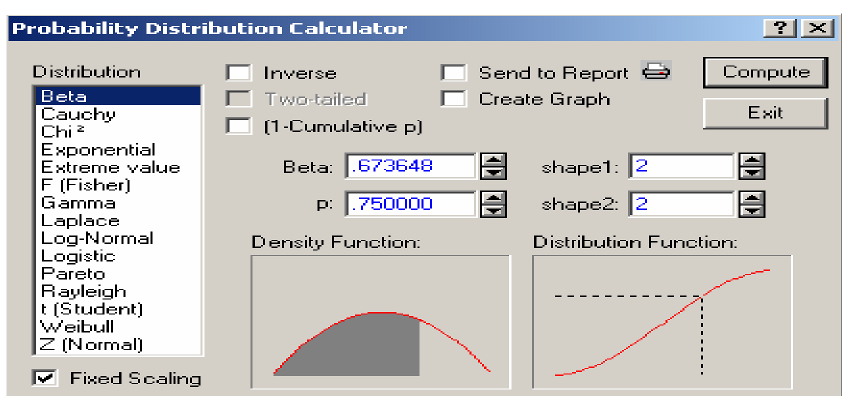

Figure 4.15 Probability Calculator for the Beta Distribution

By default, Statistica provides a **Beta** distribution with parameters $\alpha = 2$ and $\beta = 2$. To calculate $P(X < 0.673648)$, where X has a Beta distribution with parameters 2 and 2, put 0.673648 for **Beta**, click "compute" to get 0.75 for p**,** i.e., $P(X < 0.673648) = 0.75$.

Similarly for a Beta distribution with shape parameter $\alpha = 2$, and scale parameter $\beta = 3$, we can calculate $P(X < 0.673648)$, by putting 2 for the shape 1, 3 for the shape 2 and 0.673648 for **Beta** and click "compute" to get 0.894997 for *p*, i.e., $P(X < 0.673648)$ = 0.894997.

Exercises

4.1 The tread wear (in thousands of kilometers) that car owners get with a certain kind of tire is a random variable whose probability density is given by

$$f(x) = \frac{1}{30} \ e^{-x/30} \qquad 0 \le x < \infty$$

(a) Find the probability that one of these tires will last at most 18000 kilometers.
(b) Find the probability that one of these tires will last anywhere from 27000 to 36000 kilometers.
(c) Comment on the probability in (a)
if the mean time to failure is β = 10000, 20000, 30000, 40000, 50000, 60000.

4.2 A transistor has an exponential time to failure distribution with mean time to failure of β = 20,000 hours.

(a) What is the probability that the transistor fails by 30,000 hours?
(b) The transistor has already lasted 20, 000 hours in a particular application.
What is the probability that it fails by 30, 000 hours?
(c) Comment on the probability in (a)
if $\beta = 10000; 20000; 30000; 40000; 50000; 60000$.

4.3 The lifetime X (in hours) of the central processing unit of a certain type of microcomputer is an exponential random variable with parameter 0.001. What is the probability that the unit will work at least 1,500 hours?

4.4 The lifetime (in hours) of the central processing unit of a certain type of microcomputer is an exponential random variable with mean β = 1000.

(a) What is the probability that a central processing unit will have a lifetime of at least 2000 hours?
(b) What is the probability that a central processing unit will have a lifetime of at most 2000 hours

4.5 The amount of raw sugar that one plant in a sugar refinery can process in one day can be modeled as having an exponential distribution with a mean of 4 tons. What is the probability that any plant processes more than $4\,ln 2$ tons of sugar on a day?

4.6 (Johnson, R. A., 2000, 172). The amount of time that a surveillance camera will run without having to be rested is a random variable having the exponential distribution with $\lambda = 50$ days. Find the probabilities that such a camera will

(a) have to rested in less than 20 days;
(b) not have to rested in at least 60 days

4.7 (Johnson, R. A., 2000, 197). Consider a random variable having the exponential distribution with parameter $\lambda = 0.25$. Find the probabilities that

(a) it takes values more than 200;
(b) it takes values less than 300.

4.8 (Johnson, R. A., 2000, 168). If on the average three trucks arrive per hour to be unloaded at a warehouse. Find the probability that the time between the arrivals of successive trucks will be less than 5 minutes.

4.9 (Johnson, R. A., 2000, 172). The number of weekly breakdowns of a computer is a random variable having a Poisson distribution with $\lambda = 0.3$. Find the percent of the time that the interval between the breakdowns of the computer will be
(a) less than one week;
(b) at least 5 weeks.

4.10 (Johnson, R. A., 2000, 173). Given that the switchboard of a consultant's office receives on the average 0.6 calls per minute. Find the probabilities that the time between the successive calls arriving at the switchboard of the consulting firm will be
(a) less than ½ minute;
(b) more than 3 minutes.

4.11 Let Z have a standard normal distribution. Then evaluate the following:

$(a)\ P(Z \geq 1.96)$

$(b)\ P(Z \leq -1.96)$

$(c)\ P(Z \geq -1.96)$

$(d)\ P(-1.645 < Z \leq -1.28)$

$(e)\ P(-1.645 < Z \leq 1.96)$

$(f)\ P(|Z| \leq 1.96)$

$(g)\ P(|Z| > 1.96)$.

4.12 Solve the following Probability equations to find normal percentiles:

$(a)\ P(Z \geq z) = 0.05$
$(b)\ P(Z \leq -z) + P(Z \geq z) = 0.25$
$(c)\ P(Z \geq -z) + P(Z \leq z) = 0.95$
$(d)\ P(Z \leq -z) = 0.05$
$(e)\ P(Z \leq z) = 0.95$
$(f)\ P(-1.645 < Z \leq -z) = 0.15$
$(g)\ P(-1.645 < Z \leq z) = 0.90$
$(h)\ P(|Z| \leq z) = 0.95$

(*i*) $P(|Z| > z) = 0.05$.

4.13 Complete the table where the α 's are the tail probabilities of the standard normal random variable.

α	$1-\alpha$	z_{α}	$z_{1-\alpha}$	$z_{\alpha/2}$
0.005		2.575829		
			-2.326348	
		2.170090		
0.020				
				2.241403
0.050				
				1.644854
0.200				
0.250				
	0.7			
0.600				
0.750				
0.900				
0.990				

4.14 (cf. Devore, J. L., 2000, 171). Let X denote the number of flaws along a 100-m reel of magnetic tape. Suppose X has approximately a normal distribution with $\mu = 5$ and $\sigma = 5$. Calculate the probability that the number of flaws is

(a) between 20 and 30.
(b) at most 30.
(c) less than 30.
(d) not more than 25.
(e) at most 10

4.15 (Johnson, R. A., 2000, 196). If a random variable has the standard normal distribution, find the probability that it will take on a value

(a) between 0 and 2.50;
(b) between 1.22 and 2.35;
(c) between –1.33 and –0.33;
(d) between –1.60 and 1.80.

4.16 The length of each component in an assembly is normally distributed with mean 6 inches and standard deviation σ inch. Specifications require that each component be between 5.7 and 6.3 inches long. What proportion of components will pass these requirements? Comment by varying σ as

0.05 0.10 0.15 0.20 0.25 0.30 0.35 etc.

4.17 A machining operation produces steel shafts having diameters that are normally distributed with a mean of 1.005 inches and a standard deviation of 0.01 inch. Specifications call for diameters to fall within the interval 1.00 $\pm$ 0.02 inches.

(a) What percentage of the output of this operation will fail to meet specifications?
(b) Comment on the percentage in (a) if σ increases.

4.18 The weekly amount spent for maintenance and repairs in a certain company has approximately a normal distribution with a mean of \$400 and a standard deviation of \$20.

(a) If \$450 is budgeted to cover repairs for next week, what is the probability that the actual costs will exceed the budgeted amount?
(b) Comment on the probability in part (a) if μ changes, keeping σ fixed.
(c) Comment on the probability in part (a) if σ changes, keeping μ fixed.

4.19 A type of capacitor has resistance that varies according to a normal distribution with a mean of 800 megohms and a standard deviation of 200 megohms (Nelson, Industrial Quality Control, 1967, pp. 261-268). A certain application specifies capacitors with resistances between 900 and 1000 megohms. If 30 capacitors are randomly chosen from a lot of capacitors of this type, what is the probability that at least 4 of them all will satisfy the specification?

4.20 The fracture strengths of a certain type of glass average 14 (in thousands of pounds per square inche) and have a standard deviation of 1.9psi. What proportion of these glasses will have fracture strength exceeding 14.5psi?

4.21 Suppose examination scores are normally distributed with mean 60 and variance 25.

(a) What value exceeds 25% of the scores?
(b) What value is exceeded by 25% of the scores?
(c) What is the minimum score to get A+ if the top 3% students get A+?
(d) What is the maximum score leading to failure if the bottom 20% of students fails?

4.22 The life of a semi-conductor laser at a constant power is normally distributed with a mean of 7000 hours and a standard deviation of 600 hours.
(a) What is the probability that the laser fails before 5000 hours?
(b) What is the life in hours that 5% of the lasers exceed?
(c) What life (in hours) is exceeded by 5% of the lasers?

4.23 The reaction time of a driver to visual stimulus is normally distributed with a mean of 0.40 seconds and a standard deviation of 0.05 second.
(a) What is the probability that a reaction requires more than 0.50 second?
(b) What is the probability that a reaction requires between 0.4 and 0.5 second?

(c) What is the reaction time that is exceeded 90% of the time?
(d) What reaction time is exceeded 10% of the time?

4.24 The personnel manager of a large company requires job applicants to take a certain test and achieve a score of 500 or more. The test scores are distributed with mean 485 and standard deviation 30. What score is exceeded by 75% of the applicants? Assume that the test scores are normally distributed.

4.25 The personnel manager of a large company requires employees to take a certain test. The test scores are normally distributed with mean 485 and standard deviation 30. The manager will promote those applicants whose scores exceed 75^{th} percentile, and terminate those with scores less than 25^{th} percentile.

(a) What is the minimum score to have promotion in the job?
(b) What is the maximum score for getting terminated from the job?

4.26 A Company produces light bulbs whose lifetimes follow a normal distribution with mean 500 hours and standard deviation 50 hours.

(a) If a light bulb is chosen randomly from the company's output, what is the probability that its lifetime will be between 417.75 and 582.25 hours?
(b) If thirty light bulbs are chosen at random, what is the probability that more than half of them will survive more than the average lifetime?

4.27 (Devore, J. L., 2000, 164). The breakdown voltage of a randomly chosen diode of a particular time is known to be normally distributed. What is the probability that a diode's breakdown voltage is within 1 standard deviation of its mean value?

4.28 (Devore, J. L., 2000, 164). The time that it takes a driver to react to the brake lights on a decelerating vehicle is critical in helping to avoid rear-end collisions. The article "Fast-Rise Brake Lamp as a Collision-Prevention Device" (Ergonomics, 1993: 391-395) suggests that reaction time for an in-traffic response to brake signal from standard brake lights can be modeled with a normal distribution having mean value 1.25 sec and standard deviation of .46 sec. What is the probability that
(a) the reaction time is between 1.00 sec and 1.75 sec?
(b) the reaction time exceeds 2 sec?
(c) the reaction time is no more than 1.45 sec?

4.29 (Devore, J. L., 2000, 169). Suppose that the force acting on a column that helps to support a building is normally distributed with mean 15.0 kips and standard deviation 1.25 kips. What is the probability that the force
(a) is at most 17 kips?
(b) is between 10 and 12 kips?
(c) differs from 15 kips by at most 2 standard deviations?

4.30 (Johnson, R. A., 2000, 197). The burning time of an experimental rocket is a random variable having the normal distribution with mean = 4.76 seconds and standard deviation. = 0.04 second. What is the probability that this kind of rocket will burn

(a) less than 4.66 seconds?
(b) more than 4.80 seconds?
(c) anywhere from 4.70 to 4.82 seconds?

4.31 (Johnson, R. A., 2000, 172). If a random variable has the gamma distribution with $\alpha = 2$ and $\beta = 2$, find the probability that the random variable will take on a value less than 4.

4.32 (Johnson, R. A., 2000, 172). In a certain city, the daily consumption of electric power (in millions of kilowatt-hors) can be treated as a random variable having a gamma distribution with $\alpha = 3$ and $\beta = 2$. If the power plant of the city has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be adequate on any given day.

4.33 (Johnson, R. A., 2000, 171). Suppose that the lifetime of a certain kind of an emergency backup battery (in hours) is a random variable X having the Weibull distribution with $\alpha = 0.1$ and $\beta = 0.5$. Find

(a) the probability that such a battery will last more than 300 hours
(b) the probability that such a battery will last less than 380 hours
(c) the probability that such a battery will not last 100 hours.

4.34 (Johnson, R. A., 2000, 173). Suppose that the time to failure (in minutes) of certain electronic components subjected to continuous vibration may be looked upon as a random variable having the Weibull distribution with $\alpha = 1/5$ and $\beta = 1/3$. What is the probability that such a component will fail in less than 5 hours?

4.35 (Johnson, R. A., 2000, 173). Suppose that the service life (in hours) of a semiconductor is a random variable having the Weibull distribution with $\alpha = 0.025$ and $\beta = 0.500$. What is the probability that such a semiconductor will still be in operating condition after 4,000 hours?

4.36 (Johnson, R. A., 2000, 197). A mechanical engineer models the bending strength of a support beam in a transmission tower as a random variable having the Weibull distribution with $\alpha = 0.02$ and $\beta = 3.0$. What is the probability that the beam can support a load of 4.5?

4.37 (Johnson, R. A., 2000, 169). In a certain country the proportion of highway sections requiring repairs in any given year is a random variable with $\alpha = 3$ and $\beta = 2$.

(a) On the average what percentage of the highway sections requires repair in any given year?

(b) Find the probability that at most half of the highway sections will require repair in any given year?

4.38 (Johnson, R. A., 2000, 173). If the annual proportion of erroneous income tax returns filed with the IRS can be looked upon as a random variable having a beta distribution with $\alpha = 2$ and $\beta = 9$, what is the probability that in any given year there will be fewer than 105 erroneous returns?

4.39 (Johnson, R. A., 2000, 173). Suppose that the proportion of the defectives shipped by a vendor, which varies somewhat from shipment to shipment, is a random variable having the beta distribution with $\alpha = 1$ and $\beta = 4$. Find the probability that a shipment from this vendor will contain 25% or more defectives.