CHAPTER THREE
Discrete Random Variables

The following are well known discrete probability distributions.

Table 1 Discrete Probability Distributions

Distribution	Probability function	Mean	Variance
Binomial	$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, $x = 0, 1, 2, ..., n$	np	$np(1-p)$
Geometric	$P(X = x) = q^{x-1} p$, $x = 1, 2, ...$ where $q = 1-p$.	$1/p$	q/p^2
Hypergeometric	$P(X = x) = \dfrac{\binom{N_0}{x}\binom{N-N_0}{n-x}}{\binom{N}{n}}$, $x = 0, 1, ..., \min(n, N_0)$	$n\dfrac{N_0}{N}$	$n\dfrac{N_0}{N}\cdot\left(1-\dfrac{N_0}{N}\right)\cdot\dfrac{N-n}{N-1}$
Poisson	$P(X = x) = \dfrac{\lambda^x}{x!}e^{-\lambda}$, $x = 0, 1, 2, ...$	λ	λ

3.1   The Binomial Distribution

Example 3.1 An oil drilling company ventures into various locations, and their success or failure is independent from one location to another. Suppose the probability of a success at any specific location is 0.1. If a driller drills 5 locations, find the probability that there will be at least two successes.

Solution Let X be the number of successful drilling of oils in 5 locations so that $X \sim B(n = 5, \ p = 0.1)$ and

$$P(X \geq 2) = 1 - \left[P(X = 0) + P(X = 1) \right]$$

$$= 1 - \left[\binom{5}{0}(0.1)^0(0.9)^{5-0} + \binom{5}{1}(0.1)^1(0.9)^{5-1} \right]$$

$$= 0.08146.$$

Computing Binomial Probabilities Using Statistica

In Statistica, we compute binomial probability using a function called ***Binom***. This function has three arguments; x, p and n. The quantity x is the value assumed by the binomial random variable X, whereas n and p are respectively the number of trials and probability of success. Thus, to compute the binomial probability for Example 3.1, enter the numbers 0, 1, 2, 3, 4, 5 in one column, say VAR1. Next, double-click another variable, say VAR2 to obtain Figure 3.1.
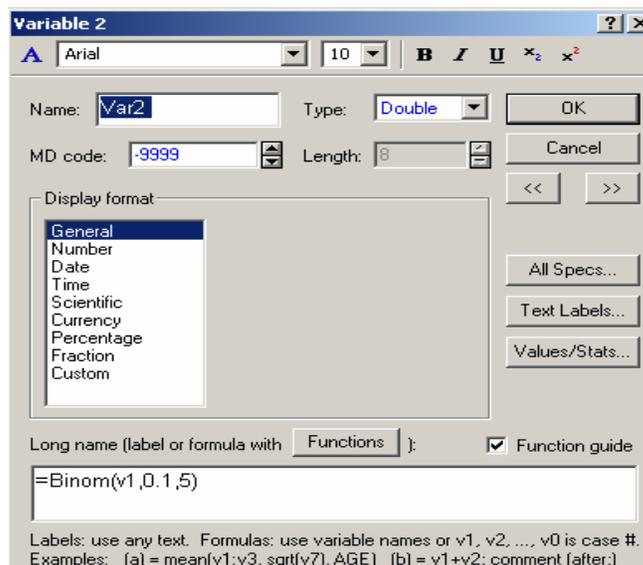

Figure 3.1 Result of Double Clicking a Variable

In the formula box at the bottom, write the formula $= Binom(v1, 0.1, 5)$ and click ***OK*** and then ***Yes*** again for the ***Expression OK Dialogue*** shown in Figure 3.2.
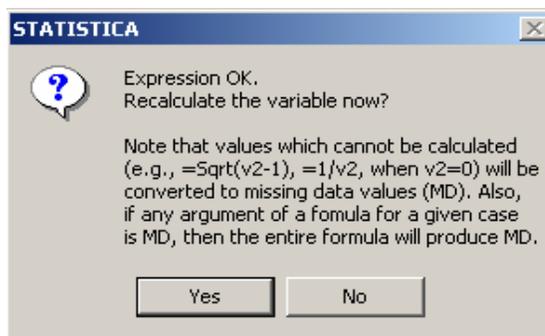

Figure 3.2 Expression OK Dialogue

You will get the probability for each number in Var1 and the Statistica will put the probability in Var2 as in Figure 3.3. Then to solve the problem in Example 3.1, add the probabilities corresponding to $x = 2$ to $x = 5$

$P(x \geq 2) = 0.0729 + 0.0081 + 0.00045 + 0.00001 = 0.0815$.

Figure 3.3 Probability Distribution of a Binomial Random Variable

To solve Example 3.1 for cumulative probabilities, one needs to go through the same steps as above and use the function $IBinom(x, p, n)$ instead of $Binom(x, p, n)$.

Instead of typing in the functions, they can be selected from the **Function Browser.** Double click the variable to get Figure 3.1. In the formula box write "=" then click **Functions** to get the function browser shown in Figure 3.4, then select **Distributions** under **Category** on the left and **Binom** under **Item** on the right and then press **Enter** to get the function $Binom(x, p, n)$ in the formula box of Figure 3.1. Next x, p and n are replaced by their values and *click OK* to get the final result in Var3.
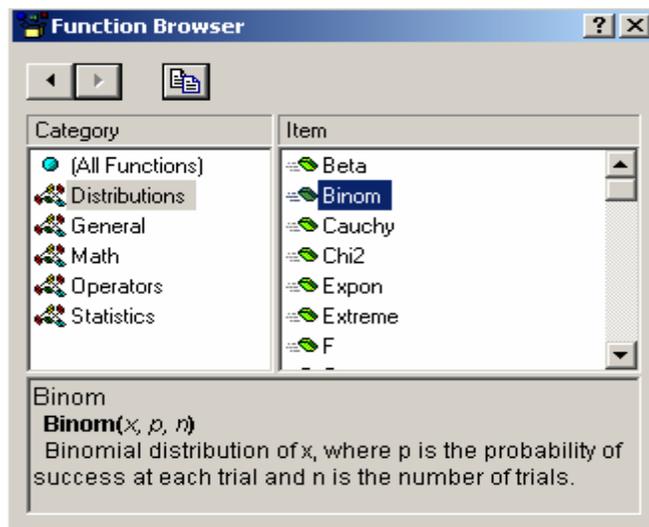

Figure 3.4 The Function Browser

Graphing the Binomial Random Variable
To plot a scatter graph for a binomial random variable, first create an appropriate data sheet and enter the values of the sample space in one column. Next, compute the binomial probabilities as above, then follow the steps:
1. Graphs/ 2D Graphs/Scatter plots
2. In 2D Scatter plots window, click Advanced
3. Choose Regular for Graph Type and set Fit to off
4. Variables (select the column containing the sample space as variable X and the column containing the probability as variable $P(x)$)/ OK
5. OK.

3.2 The Geometric Distribution

Example 3.2 A manufacturer uses electrical fuses in an electronic system. The fuses are purchased in large lots and tested sequentially until the first defective fuse is observed. Assume that the lot contains 10% defective fuses. What is the probability that the

(a) first defective fuse is observed on the first test?
(b) first defective fuse is observed on the second test?
(c) first defective fuse is observed on the third test?

Solution (a) 0.10 (b) $(0.90)(0.10)$ (c) $(0.90)^2 (0.10)$.

Computing Geometric Probabilities Using Statistica
In Statistica, we compute geometric probabilities using a function called *Geom.* This function has two arguments: x and p. The quantity x is the value assumed by the geometric random variable X, and p is the probability that a particular event (e.g., success) will occur. Thus, $P(X = x) = Geom(x, p) = p(1-p)^x$

is used to compute the geometric probability for part (b) in Example 3.2. Double-click any variable, say VAR2 to obtain the Figure 3.5.
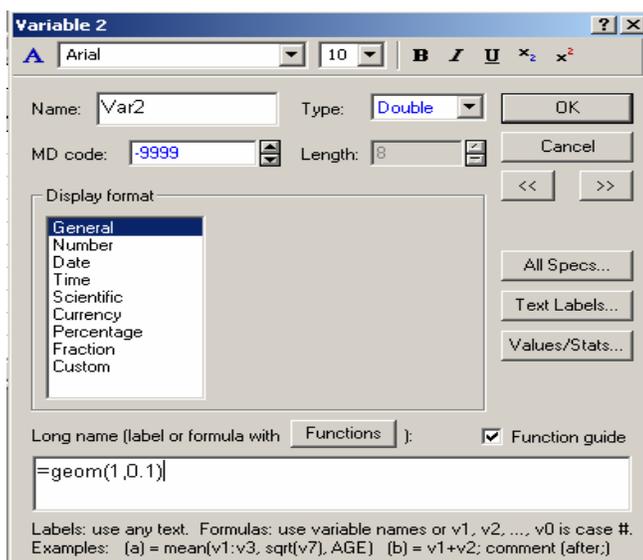

Figure 3.5 Result of Double clicking a variable

Write the formula $= Geom(1,0.1)$ in the formula box at the bottom, and click *OK* and then *Yes* again for the *Expression OK Dialogue* as shown in Figure 3.2 to get the answer (0.09) in all cases in that variable (VAR2) as can be seen in Figure 3.6.

Figure 3.6 Result of "=Geom(1, 0.1)"

Note: We can also compute geometric probabilities using the *Function Browser* as in the case of binomial probabilities. Also cumulative probability can be calculated using the function $IGeom(x, p)$.

3.3 The Hypergeometric Distribution

Example 3.3 Suppose that a random sample of size 2 is selected without replacement, from a lot of 100 laser printers and it is known that 5% of the items in the lot are defective. What is the probability that
(a) none of them is defective?
(b) one of them is defective?
(c) both are defective?

Solution:

$$(a)\ \frac{\binom{5}{0}\binom{95}{2}}{\binom{100}{2}} = 0.902 \qquad (b)\ \frac{\binom{5}{1}\binom{95}{1}}{\binom{100}{2}} = 0.096 \qquad (c)\ \frac{\binom{5}{2}\binom{95}{0}}{\binom{100}{2}} = 0.002$$

3.4 The Poisson Distribution

Example 3.4 The number of failures of a testing instrument from contamination particles on the product is a Poisson random variable with a mean of 0.02 failures per hour.
   (a) What is the probability that the instrument does not fail in an 8-hour shift?
   (b) What is the probability of at least one failure in 30 minutes?

Solution

(a) Let X = number of failures of the testing instrument per 8-hour hour. Then the expected number of failures in an 8-hour shift is given by

$$\lambda = 0.02(8) = 0.16$$

so that

$$P(X = 0) = e^{-\lambda} = e^{-0.16} = 0.852.$$

(b) Let X = number of failures of the testing instrument in 30 minutes. Then the expected number of failures in 30 minutes is given by

$$\lambda = 0.02(30/60) = 0.01$$

so that

$$P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-\lambda} = 1 - e^{-0.01} = 0.01.$$

Computing Poisson Probabilities Using Statistica

In Statistica, we compute Poisson probability using a function called *Poisson*. This function has two arguments: x and lambda (λ). The quantity x is the value assumed by the Poisson random variable X, and lambda (λ) is the expected value of x. Thus,

$$P(X = x) = Poisson(x, lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

The Poisson probability can be computed in the same way as for the binomial and geometric distributions. To compute the Poisson probability $P(X \geq 1) = 1 - P(X = 0)$ for part (b) in Example 3.4, first compute lambda $(\lambda) = 0.01$. Double click any variable, say VAR2, to obtain Figure 3.5 and in the formula box at the bottom, write the formula "= 1-Poisson (0, 0.01)." Click *OK* and then **Yes** again for the **Expression OK Dialogue** as shown in Figure 3.2, to get (0.009955) as in Figure 3.7. Cumulative probabilities can also be calculated using the function $IPoisson(x, lambda)$.

	1 Var1	2 Var2	3 Var3	4 Var4
1		0.00995		
2		0.00995		
3		0.00995		
4		0.00995		
5		0.00995		
6		0.00995		
7		0.00995		
8		0.00995		
9		0.00995		
10		0.00995		

Data: Spreadsheet1* (10v by 10c)

Figure 3.7 Result of "=1-Poisson(0, 0.01)"

Note that we can also compute the Poisson probabilities using the **Function Browser** as done for the case of binomial.

Exercises

3.1 (Johnson, R. A., 2000, 139). If the probability is 0.20 that a downtime of an automated production process will exceed 2 minutes, find the probability that 3 of 8 downtimes of the process will exceed 2 minutes.

3.2 (Johnson, R. A., 2000, 139). If the probability that a fluorescent light has a useful life of at least 500 hours is 0.85, find the probabilities that among 20 such lights

(a) 18 will have a useful life of at least 500 hours.
(b) at least 15 will have a useful life of at least 500 hours.
(c) at most 10 will not have a useful life of at least 500 hours.

3.3 (Johnson, R. A., 2000, 125). It is known that 5% of the books bound at a certain bindery have defective bindings. Find the probability that at least 20 of 100 books bound by this bindery will have defective bindings.

3.4 (cf. Devore, J. L., 2000, 123). Suppose that 20% of all copies of a particular textbook fail a certain binding strength test. Let X denote the number among 15 randomly selected copies that fail the test. Then X has a binomial distribution with $n = 15, p = 0.2$.

(a) Complete the probability and cumulative probability distribution for the number of failures.
(b) Draw the probability and cumulative probability histograms.
(c) Find the probability that at most 8 fail the test.
(d) Find the probability that exactly 8 fail the test.
(e) Find the probability that at least 8 fail.
(f) Find the probability that between 4 and 7, inclusive, fail the test.

3.5 (cf. Devore, J. L., 2000, 123). An electronic manufacturer claims that 10% of its power supply units need service during the warranty period. To investigate this claim, technicians at a testing laboratory purchase 20 units and subject each unit to accelerated testing to simulate use during the warranty period.

(a) Find the complete probability and cumulative probability distributions for the number of units that need repair during the warranty period.
(b) Draw the probability and cumulative probability histograms.
(c) Find the probability that at most 6 need repair during the warranty period.
(d) Find the probability that exactly 12 need repair during the warranty period.
(e) Find the probability that between 5 and 10, inclusive, need repair during the warranty period.

3.6 (cf. Devore, J. L., 2000, 125). Compute the following binomial probabilities
(a) $b(3;8;0.6)$.

(b) $P(3 < X \leq 5)$ when $n = 8$ *and* $p = 0.6$.

(c) $P(X < 3)$ when $n = 12$ *and* $p = 0.1$.

(d) $P(X \geq 4)$ when $n = 10$ *and* $p = 0.3$.

(e) $P(X \geq 15)$ when $n = 30$ *and* $p = 0.3$

3.7   (cf. Devore, J. L., 2000, 125). When circuit boards used in the manufacture of compact disc players are tested, the long run percentage of defectives is 5%. Let X = number of defective boards in a random sample of size $n = 35$. Determine:

(a) $P(X \geq 10)$.

(b) $P(X \leq 20)$.

(c) $P(8 \leq X \leq 25)$.

(d) What is the probability that none of the 35 boards are defective?

(e) Calculate the expected value and standard deviation of X.

3.8   (cf. Devore, J. L., 2000, 125-126). A company that produces fine crystal knows from experience that 10% of its goblets have cosmetic flaws and must be classified as "seconds."

(a) Among six randomly selected goblets, how likely is it that only at least one is a second?

(b) Among six randomly selected goblets, what is the probability that at least two are seconds?

(c) If goblets are examined one by one, what is the probability that at most 5 of six are seconds?

3.9   (cf. Devore, J. L., 2000, 126). Suppose that only 20% of all drivers come to a complete stop at an intersection having flashing red lights in all directions when no other cars are visible. What is the probability that, of 20 randomly chosen drivers coming to an intersection under these conditions:

(a) at most 7 will come to a complete stop?

(b) more than 6 will come to a complete stop?

(c) at least 8 will come to a complete stop?

(d) not all 20 will come to a stop?

3.10   (cf. Walpole, R. E, et. al, 2002, 134). In a certain manufacturing process it is known that, on the average, 1 in every 100 items is defective. What is the probability that

(a) the fifth item inspected is the first defective item found?

(b) at least four defective items are checked before the first non defective item?

(c) at most four defective items are checked before the first non defective item?

3.11   (cf. Walpole, R. E, et. al, 2002, 135). At Busy time a telephone exchange is very near capacity, so callers have difficulty placing their calls. It may be of interest to know the number of attempts necessary in order to gain a connection. Suppose that we let $p = 0.05$ be the probability of the connection during busy time. What is the probability that

(a) 5 attempts are necessary for a successful call?
(b) 12 attempts are necessary for a successful call?
(c) 20 attempts are necessary for a successful call?

3.12   (cf. Walpole, R. E, et. al, 2002, 139). The probability that a student pilot passes the written test for a private pilot's license is 0.7. Find the probability that the student will pass the test

(a) on the third try.
(b) on the seventh try.
(c) on the ninth try.

3.13   (Johnson, R. A., 2000, 139). A basketball player makes 90% of his free throws. What is the probability that he will miss for the first time on the seventh shot?

3.14   (cf. Walpole, R. E, et. al, 2002, 137). During a laboratory experiment the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is the probability that

(a) 6 particles enter the counter in a given millisecond?
(b) more than 8 particles enter the counter in a given millisecond?
(c) no less than 2 particles enter the counter in a given millisecond?
(d) less than 10 particles enter the counter in a given millisecond?

3.15   (cf. Walpole, R. E, et. al, 2002, 137). Ten is the average number of oil tankers arriving each day at a certain port city. The facilities at the port can handle at most 15 tankers per day. What is the probability that that on a given day tankers have to be turned away?

3.16   (Walpole, R. E, et. al, 2002, 139). On average, a certain intersection results in 3 traffic accidents per month. What is the probability that for any given month at this intersection

(a) exactly 5 accidents will occur?
(b) less than 3 accidents will occur?
(c) at least 2 accidents will occur?

3.17   (Walpole, R. E, et. al, 2002, 139). A secretary makes 2 errors per page on average. What is the probability that on the next page, he will make

(a) 4 or more errors?
(b) no errors?
(c) less than 10 errors?

3.18   (Walpole, R. E, et. al, 2002, 139). A certain area in the eastern United States is, on average, hit by 6 hurricanes per year. Find the probability that for a given year that area will be hit by

(a) fewer than 8 hurricanes.
(b) anywhere from 4 to 12 hurricanes.
(c) more than 10 hurricanes.

3.19 (Walpole, R. E, et. al, 2002, 139). The average number of field mice per acre in a wheat field is estimated to be 12. Find the probability that on a given acre

(a) fewer than 7 field mice are found.
(b) no less than 10 field mice are found.
(c) no fewer than 5 field mice are found.

3.20 (Johnson, R. A., 2000, 128). If a bank receives on the average 6 bad checks per day, what are the probabilities that it will receive

(a) 4 bad checks in any given day?
(b) 10 bad checks over any two consecutive days?

3.21 (Johnson, R. A., 2000, 128). In the inspection of tinplate produced by a continuous electrolytic process, 0.2 imperfections are spotted on the average per minute. Find the probabilities of spotting

(a) one imperfection in 3 minutes.
(b) at least 2 imperfections in 5 minutes.
(c) at most one imperfection in 15 minutes.

3.22 (Johnson, R. A., 2000, 131). Given that the switchboard of a consultant's office receives on the average 0.6 calls per minute. Find the probabilities that

(a) in a given minute, there will be at least 1 call.
(b) in a given minute, there will be at least 5 call.

3.23 (Johnson, R. A., 2000, 131). At a checkout counter customers arrive at an average of 1.5 per minute. Find the probabilities that

(a) at most 4 will arrive in any given minute.
(b) at least 3 will arrive during an interval of 2 minutes.
(c) at most 15 will arrive during an interval of 6 minutes.

3.24 (Johnson, R. A., 2000, 129). If on the average three trucks arrive per hour to be unloaded at a warehouse, find the probability that at most 20 will arrive during an 8-hour day shift.

3.25 (Johnson, R. A., 2000, 140). The number of weekly breakdowns of a computer is a random variable having a Poisson distribution with $\lambda = 0.3$. What is the probability that the computer will operate without a breakdown for 2 consecutive weeks?

3.26 In a shipment of 50 hard disks, five are defective. If four of the disks are randomly selected for inspection, what is the probability that more than 2 are defective?

3.27 A foundry ships engine blocks in lots of 20. Three items are selected and tested. If the lot actually contains five defective items, find the probability that there will be at least 2 defective blocks in the sample?

3.28 During the course of an hour 1000 bottles of soft drinks are filled by a particular machine. Each hour a sample of 20 bottles is randomly selected and the number of ounces of soft drink bottle is checked. Suppose that during a particular hour 100 underfilled bottles are produced. What is the probability that at least 3 underfilled bottles will be among those sampled?

3.29 Twenty microprocessor chips are in stock. Three have etching errors that cannot be detected by the naked eye. Five chips are selected and installed in field equipment. Find the probability that at least one chip with an etching error will be chosen.

3.30 Production line workers assemble 15 automobiles per hour. During a given hour, four are produced with improperly fitted doors. Three automobiles are selected at random and inspected. Find the probability that at most one will be found with improperly fitted doors.