KFUPM Mathematical Sciences	Term 042 STAT 212 Quiz# 3	Date: 19/3/2005 Duration: 10 minutes

Name: ID#: Serial#: Section#: 1 2 3

Write neatly and eligibly

The Department of Weights and Measures in a southern state has the responsibility for making sure that all commercial weighing and measuring devices are working properly. For example, when a gasoline pump indicates that 1 gallon has been pumped, it is expected that 1 gallon of gasoline will actually have been pumped. The problem is that there is variation in the filling process. The state's standards call for the standard deviation to be less than 0.01 gallons. Recently, the department came to a gasoline station and filled 10 cans until the pump read 1.0 gallon. They then measured precisely the amount of gasoline in each can. The following data were recorded:

0.991	0.962	1.007	1.038	1.036
1.052	0.934	0.993	1.033	0.967

Based on these data what should the Department of Weights and Measures conclude if they wish to test using a 0.05 level of significance?

ANSWER:

With respect to the variation requirement that specifies that the standard deviation should be less than 0.01 gallon, we need to use a test for a single population variance. The appropriate null and alternative hypotheses are:

$$H_o : \sigma^2 \geq 0.0001$$
$$H_A : \sigma^2 < 0.0001$$

Note that we have converted from standard deviation to variances since there is no test for standard deviation directly. In this situation, the test statistic is a chi-square value computed as follows:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(10-1).0388^2}{.01^2} = 135.49.$$

This test statistic can be compared to the chi-square value from the table with 10-1 = 9 degrees of freedom and a one-tail area of 0.95. That value is 3.325. Since $\chi^2 = 135.49 > 3.325$, the null hypothesis is NOT rejected. This means that the variation standard is being exceeded based on these data. The gasoline station will need to get the pump checked out and the variation of fill around the mean reduced.

With My Best Wishes

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Mathematical Sciences

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There are two major companies that provide SAT test tutoring for high school students. At issue is whether Company 1 that has been in business for the longer time provides better results than Company 2, the newer company. Specifically of interest is whether the mean increase in SAT scores for students who have already taken the SAT-test one time is higher for Company 1 than for Company 2. A test of this is to be conducted using a 0.05 level of significance. Two random samples of students are selected. The first group uses the tutoring services of Company 1 and the second uses Company 2's services. The following data reflect the number of points higher (or lower) that the students scored on the SAT-test after taking the tutoring.

Based on these two small independent samples with unknown population variances, if you want to test that the students who use Company 1 score higher on average than students who use Company 2, what can you assume about the equality of the two population variances?

Company1	98	60	-14	80	55	71	37	41
Company2	65	11	30	52	27	16	27	47

ANSWER:

This test calls for us to test whether the population means are equal. Since the sample sizes are small and the population variances are unknown, the t-distribution should be employed if we assume that the populations are normally distributed and the populations have equal variances. We can test for the equal variance assumptions as follows.

The following null and alternative hypotheses would be appropriate in this situation:

$$H_o : \sigma_1^2 = \sigma_2^2$$
$$H_o : \sigma_1^2 \neq \sigma_2^2 .$$

This will be a two-tailed test since we are testing to see whether a difference exists and are not predicting which company will have more or less variability.

We first need to compute the sample variances for each sample using:

$$s^2 = \frac{\sum (x - \bar{x})^2}{n-1} .$$

The following variances are computed from the sample data:

$$s_1^2 = 1145.43 \text{ and } s_2^2 = 345.70 .$$

In a two-tailed test of variances, the test statistic is an F-value that is formed by the ratio of the two sample variances and by placing the larger sample variance in the numerator:

$$F = \frac{s_1^2}{s_2^2} = \frac{1145.43}{345.7} = 3.3134 .$$

We then compare this value to a critical value from the F-distribution table. Since this is a two tail test with alpha = .10, we use the table with 0.05 in the upper-tail of the F-distribution.

Two sets of degrees of freedom are used with the F-distribution. Across the top of the F-table we look for degrees of freedom corresponding to n-1 where n is the sample size associated with the sample variance that was placed in the numerator of the F-test statistic. In our case, n = 8, so the degrees of freedom is 8-1 = 7 for the numerator. The degrees of freedom down the side in the F table is n-1 where n corresponds to the sample size for the sample variance in the denominator of the F-test statistic. In our case that would be n = 8. Thus, the denominator degrees of freedom are 8-1 = 7. Then the critical F from the .05 table with 7 and 7 degrees of freedom is 3.787. The decision rule is:

If F calculated > 3.787 reject H_o, otherwise do not reject.

Since our calculated F-test statistic is F = 3.3134 < 3.787, we do not reject the null hypothesis. Thus, based on the sample information, we have no basis for believing that there is a difference in the two companies with respect to population variance.

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