Chapter 8

1. Testing hypotheses about μ

a. For large sample or known σ, z values are used.

b. For small sample and unknown σ, t values are used.

The test statistics are

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \quad \text{or} \quad t_{n-1} = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$ will be compared with

a. z_α or t_α for one tailed hypotheses.

b. $z_{\alpha/2}$ or $t_{\alpha/2}$ for two tailed hypotheses.

2. Testing hypotheses about *p*

The test statistic

$$z = \frac{\bar{p} - p}{\sqrt{p(1-p)/n}} \quad \text{where } \bar{p} = \frac{x}{n}$$

Chapter 9

1. Testing hypotheses about the match paired

The i^th paired difference is $d_i = x_{1i} - x_{2i}$

Test statistic is $$t_{n-1} = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}}$$

where $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$ and $s_d = \sqrt{\sum_{i=1}^{n}(d_i - \bar{d})^2 / (n-1)}$.

2. Testing difference between two independent population means

a. If σ_1 and σ_2 known or n_1 and $n_2 \geq 30$

The test statistics is

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2 / n_1) + (\sigma_2^2 / n_2)}}$$

b. If σ_1 and σ_2 unknown or n_1 and $n_2 \geq 30$, the test statistic is

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(s_1^2 / n_1) + (s_2^2 / n_2)}}$$

c. If σ_1 and σ_2 unknown or n_1 or $n_2 < 30$ (assuming equal σ's)

$$t_{n_1+n_2-2} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{(1/n_1) + (1/n_2)}}$$

where

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

d. If σ_1 and σ_2 unknown or n_1 or $n_2 < 30$ (assuming unequal σ's), the test statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(s_1^2 / n_1) + (s_2^2 / n_2)}}$$

with degree of freedoms

$$df = \frac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{\left(s_1^2/n_1\right)^2/(n_1 - 1) + \left(s_2^2/n_2\right)^2/(n_2 - 1)}$$

3. To test hypotheses about difference between two independent proportions

The test statistic is

$$z = \frac{(\bar{p}_1 - \bar{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,(1 - \bar{p})\,((1/n_1) + 1/n_2)}}$$

where $\bar{p} = \frac{n_1 \bar{p}_1 + n_2 \bar{p}_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$

Chapter 10

1. Testing hypotheses about σ

The test statistic is

$\chi^2 = (n-1)s^2 / \sigma_0^2$

which has a χ^2 distribution with $df = n - 1$ and σ_0^2 is hypothetical.

2. Testing hypotheses about $\sigma_1^2 - \sigma_2^2$

The test statistic is

$F_0 = s_1^2 / s_2^2$ with $df_1 = n_1 - 1$ and $df_2 = n_2 - 1$.

Chapter 12

1. For Goodness-of-fit test, the statistic is

$$\chi^2 = \sum \frac{(o_i - e_i)^2}{e_i}$$ with k – 1 degrees of freedom

o_i = Observed cell frequency

e_i = Expected cell frequency

2. For Test of Independence, the test statistic is

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c} \frac{(o_{ij} - e_{ij})^2}{e_{ij}}$$ where

e_{ij} = (i^th row total)(j^th column total) /(sample size)

Chapter 13

1. Sample correlation coefficient

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$$

$$= \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{\left[n\sum x^2 - (\sum x)^2\right]\left[n\sum y^2 - (\sum y)^2\right]}}$$

2. For testing hypotheses about ρ, the test statistic

$t_{n-2} = r \Big/ \sqrt{(1-r^2)/(n-2)}$ has df=n-2

3. Estimated regression model

$$\hat{y}_i = b_0 + b_1 x$$

4. The Least Square Estimates are

$$b_1 = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sum (x-\bar{x})^2} = \frac{\sum xy - \left(\sum x \sum y\right)/n}{\sum x^2 - \left(\sum x\right)^2/n}$$

and

$$b_0 = \bar{y} - b_1 \bar{x}$$

5. Total Sum of Squares

$$SST = \sum (y-\bar{y})^2 = \sum_1^n y_i^2 - n\bar{y}^2$$

6. Regression & Error Sum of Squares

$$SSR = \sum (\hat{y}-\bar{y})^2 = b_1\left(\sum xy - \left(\sum x \sum y\right)/n\right)$$

$$SSE = \sum (y-\hat{y})^2 = SST - SSR$$

7. Coefficient of Determination

R- Squared = $R^2 = \frac{SSR}{SST}$

$$R^2 = \frac{SST - SSE}{SST} = 1 - \frac{SSE}{SST} = r^2$$

8. Standard Error of the Estimate

$$s_\varepsilon = \sqrt{SSE/(n-k-1)}$$

9. Standard Deviation of the Slope

$$s_{b_1} = \frac{s_\varepsilon}{\sqrt{\sum (x-\bar{x})^2}} = \frac{s_\varepsilon}{\sqrt{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}}$$

For testing H_0: $\beta_1 = 0$ vs. H_1: $\beta_1 \neq 0$

10. The test statistic & C.I. for the slope

$$t_{n-2} = \frac{b_1 - \beta_1}{s_{b_1}} \ \& \ b_1 \pm t_{\alpha/2} s_{b_1}$$

11. C.I. for the mean of y given a particular x_p

$$\hat{y} \pm t_{\alpha/2} s_\varepsilon \sqrt{\frac{1}{n} + \left[(x_p - \bar{x})^2 \Big/ \sum (x-\bar{x})^2\right]}$$

12. C.I. estimate for an Individual value of y given a particular x_p

$$\hat{y} \pm t_{\alpha/2} s_\varepsilon \sqrt{1 + \frac{1}{n} + \left[(x_p - \bar{x})^2 \Big/ \sum (x-\bar{x})^2\right]}$$

Chapter 14

1. Estimated multiple regression model

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_k x_k$$

2. Two variable model is

$\hat{y} = b_0 + b_1 x_1 + b_2 x_2$ & $e_i = y_i - \hat{y}_i$

is Errors (residuals) from regression model

3. Proportion of variation in *y* explained by *x* adjusted for the number of *x* variables used

$$R_A^2 = 1 - \left(1 - R^2\right)\left(\frac{n-1}{n-k-1}\right)$$

4. For testing $\beta_1 = \beta_2 = \ldots = \beta_k$ Test statistic

$$F = \frac{SSR/k}{SSE/(n-k-1)} = \frac{MSR}{MSE}$$

with $df_1 = k$ and $df_2 = n-k-1$

5. For testing H_0: $\beta_i = 0$ vs. H_A: $\beta_i \neq 0$

The test statistic & C.I. for the slope β_i are

$t_{n-k-1} = \frac{b_i - 0}{s_{b_i}}$ & $b_i \pm t_{\alpha/2} s_{b_i}$, respectively.

5. The estimate of the standard deviation of the regression model is

$s_\varepsilon = \sqrt{SSE/(n-k-1)} = \sqrt{MSE}$ & $VIF_j = \frac{1}{1-R_j^2}$

is the Variance Inflationary Factor (VIFj)

Chapter 15

1. Simple Index number formula & Unweighted aggregate price index formula (respectively)

$$I_t = \frac{y_t}{y_0} 100 \ \& \ I_t = \frac{\sum p_t}{\sum p_0} 100$$

2. Paasche & Laspeyres Weighted Aggregate Price Indexes (respectively)

$$I_t = \frac{\sum q_t p_t}{\sum q_t p_0} 100 \ \& \ I_t = \frac{\sum q_0 p_t}{\sum q_0 p_0} 100$$

3. Deflation formula

$$y_{adj_t} = \frac{y_t}{I_t} 100$$

4. Forecasting formula & Residual formula are

$F_t = \hat{y} = b_0 + b_1 t$ & $e_t = y_t - F_t$ respectively.

5. Mean Square Error & Mean Absolute Deviation are (respectively)

$$MSE = \sum (y_t - F_t)^2 / n \ \& \ MAD = \sum |y_t - F_t| / n$$

6. For testing H_0: ρ=0 vs. H_A: ρ≠0

Durbin-Watson Test statistic

$$d = \sum_{t=1}^{n} (e_t - e_{t-1})^2 \Big/ \sum_{t=1}^{n} e_t^2$$

7. Multiplicative Time-Series Model

$$y_t = T_t \times S_t \times C_t \times I_t$$

T_t = Trend value S_t = Seasonal value
C_t = Cyclical value I_t = Irregular (random) value

8. Ratio-to-Moving Average formula & Deseasonalizing formula (respectively)

$$S_t \times I_t = \frac{y_t}{T_t \times C_t} \ \& \ T_t \times C_t \times I_t = \frac{y_t}{S_t}$$

9. Single Exponential Smoothing Model

$F_{t+1} = F_t + \alpha(y_t - F_t) = \alpha y_t + (1-\alpha)F_t$ where
α: smoothing constant.

10. Double Exponential Smoothing Model

$$C_t = \alpha y_t + (1-\alpha)(C_{t-1} + T_{t-1})$$

$$T_t = \beta(C_t - C_{t-1}) + (1-\beta)T_{t-1} \ \& \ F_{t+1} = C_t + T_t$$

α: Constant-process smoothing constant
β: Trend-smoothing constant