# KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
# DEPARTMENT OF MATHEMATICAL SCIENCES
# DHAHRAN, SAUDI ARABIA

STAT 319: PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS

Major Exam No. II, Fall Semester (042)
Time: 0730 - 0900 pm, Tuesday 5th April, 2005

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Serial # Surname: ID# Section #

You are allowed to use electronic calculators and other reasonable accessories that help write the exam. Try to define events, formulate problem and solve.

1. Do **not** keep your **mobile** with you during the exam, turn off and leave it aside.
2. **Write** your name, ID number, Section, and serial number (optional) on the cover page.
3. **Check** that the exam paper has _8_ questions and 6 pages.
4. You can **only** ask **questions** if you feel there is any mistake in the exam needing announcement for correction.

| Q | Full Marks | Marks Obtained | Strengths/ Weakness |
|---|---|---|---|
| 1 | 7 | | |
| 2 | 7 | | |
| 3 | 9 | | |
| 4 | 4 | | |
| 5 | 5 | | |
| 6 | 4 | | |
| 7 | 4 | | |
| Total | 40 | | |

1. Suppose that the lifetime of a certain kind of emergency backup battery (in hours) is a random variable with mean 300 hours. Assume that the lifetime of a battery follows **exponential distribution**.

(a) What is the probability that such a battery will last **more than** 300 hours?
(b) What lifetime is **exceeded by** 5% of the batteries? [4 +3 = 7 Marks]

2. If 5% of the memory chips made in a certain plant are **defective**, what is the probability that

(a) in a lot of 10 randomly chosen chips for inspection exactly 5 will be defective?
(b) the first defective chip will be inspected in the $4^{th}$ trial?
(c ) the first defective chip will be inspected **on or before** the $3^{rd}$ trial? [3+2 +2 =7 Marks]

3. If a 1-gallon can of paint covers on the **average** 513.3 square feet with a **standard deviation** of 31.5 square feet, what is the probability that

(a) a 1-gallon can of paint will cover **more than** 565.1175 feet assuming that the coverage area by a 1-gallon can of paint follows a **normal distribution**?
(b) the **mean area** covered by a sample of 36 of these 1-gallon cans will be **more than** 521.93625 square feet? [4+5 = 9 Marks]

4. The number of errors per 1000 lines of computer code is described by a **Poisson distribution** with a **mean** of five errors per 1000 lines of code. What is the probability of eight errors in 2000 lines of computer code? [4 Marks]

5. Suppose that the silicon wafers used in making a particular microcircuit have a final chip that contains 95% **non-defectives**. A sample of 36 chips is taken. What is the probability that **at most** 6 of them are **defectives**? [5 Marks]

6. The number of trucks arriving to be unloaded at a receiving dock is a Poisson Process with average number of trucks arrived **per hour** is 3. What is the probability that **time between arrivals** of successive trucks will be at least **0.8 hour**? [4 Marks]

7. A quality control inspector **accepts** shipments whenever a sample **without replacement** of size 5 contains **no defectives** and he rejects otherwise.

(a) What is the probability that he will **accept** a ("bad") shipment of 50 items containing 20% defectives? [2 Marks]

(b) What is the probability that he will **reject** a ("good") shipment of 100 items in which only 2% are defective? [2 Marks]

### *FORMULAE FOR STAT 319*

## *C. Discrete Probability Distributions*

C.0a $\mu = E(Y) = \sum yp(y)$, $(p89)$

C.0b $E\left(Y^2\right) = \sum y^2\ p(y)$, $\sigma^2 = E(Y-\mu)^2 = E\left(Y^2\right) - \mu^2$, $(p96)$

| | *Probability Density function* $p(x)$ | *Mean* $(\mu)$ *and Variance* $(\sigma^2)$ |
|---|---|---|
| *C.1* | *The Binomial Distribution:* $B(n,p)$ *(p119)* $f(y) = \binom{n}{y} p^y (1-p)^{n-y}$, $y = 0, 1, \ldots, n$, *where* $\binom{n}{y} = \frac{n(n-1)\ldots(n-y+1)}{y!}$. | $\mu = E(Y) = np$ $\sigma^2 = V(Y) = np(1-p)$. |
| *C.2* | *The Hypergeometric Distribution (p128)* $f(y) = \frac{\binom{K}{y}\binom{N-K}{n-y}}{\binom{N}{n}}$, $y = 0, 1, \ldots,$ | $\mu = E(Y) = n\ (K/N)$ |
| *C.3* | *The Poisson Distribution (p136)* $f(y) = \frac{e^{-\lambda t}(\lambda t)^y}{y!}$, $y = 0, 1, \ldots$ | $\mu = E(Y) = \lambda t$ $\sigma^2 = V(Y) = \lambda t$ |

## *D. Continuous Probability Distributions*

For a continuous random variable $Y$ with pdf $f(y)$

$D.0$ $\int_{-\infty}^{\infty} f(y)dy = 1$; $P(a < Y < b) = \int_a^b f(y)dy$; $P(Y \le u) = \int_{-\infty}^{u} f(y)dy$

$D.0a$ $\mu = E(Y) = \int_{-\infty}^{\infty} yf(y)dy$, $(p89)$

$D.0b$ $E\left(Y^2\right) = \int_{-\infty}^{\infty} y^2 f(y)dx$, $\sigma^2 = E(Y-\mu)^2 = E\left(Y^2\right) - \mu^2$, $(p96)$

| | Probability Density function | Mean and Variance |
|---|---|---|
| D.1 | The Normal Distribution: $N(\mu,\sigma^2)$ | Mean = $E(Y)=\mu$<br>Variance = $V(Y)=\sigma^2$ |
| D.2 | The Exponential Distribution<br>$f(y)=\dfrac{1}{\beta}\,e^{-y/\beta},\ 0<y$ | Mean = $E(Y)=\beta$<br>Variance = $V(Y)=\beta^2$ |
| D.3 | Waiting Time Distribution<br>$f(t)=\lambda e^{-\lambda t},\ 0<t$ | Mean = $E(T)=1/\lambda$<br>Variance = $V(T)=1/\lambda^2$ |
| D.4 | The Gamma Distribution<br>$f(y)=\dfrac{1}{\beta^\alpha\Gamma(\alpha)}\,y^{\alpha-1}\,e^{-y/\beta},\ 0<y,\ 0<\alpha,\ 0<\beta$ | Mean = $E(Y)=\alpha\beta$<br>Variance = $V(Y)=\alpha\beta^2$ |

## *E. Sampling Distributions*

*E.1 Suppose that $Y$ has a distribution with mean $\mu$ and variance $\sigma^2$. Additionally if the distribution is **normal** then* $\dfrac{\sum Y-n\mu}{\sqrt{n\sigma^2}}=\dfrac{\bar{Y}-\mu}{\sqrt{\sigma^2/n}}=Z$.

*E.2 Suppose that $Y$ has a distribution with mean $\mu$ and variance $\sigma^2$. However if the distribution is not normal but $n\geq 30$, then* $\dfrac{\sum Y-n\mu}{\sqrt{nS^2}}=\dfrac{\bar{Y}-\mu}{\sqrt{S^2/n}}\approx Z$ *(p210). This is known as **Central Limit Theorem**. Note that $\sigma^2$ is estimated by $s^2$ (SLLN).*

*E.3 The **Student t statistic** is defined by* $T=\dfrac{\bar{Y}-\mu}{S/\sqrt{n}}$*, with* $\nu=n-1$ *(p220)*

*E.4 **The Sampling Distribution of the Proportion** (p258)*

$$\frac{Y-np}{\sqrt{np(1-p)}}=\frac{Y/n-p}{\sqrt{p(1-p)/n}}\approx Z$$