KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
DHAHRAN, SAUDI ARABIA

STAT 319: PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS

Major Exam No. I, Fall Semester (042)
Time: 0900 - 1030 pm, Monday14th March, 2005

Instructors: Anwar Joarder, Musawar Malik, Mohammad Omar, Essam AlSawie, Ibrahim Rahimov

Please circle your instructor's name. You are allowed electronic calculators and other reasonable accessories (not a mobile of course) that help write the exam. Try to define events, formulate problem and solve.

Time Management: For **each mark** you have 2 minutes making a total of 80 minutes and further 10 minutes for reviewing the exam.

Student Surname: ID# Section #

Q	Full Marks	Marks Obtained	Strengths/ Weakness
1	3		
2	2		
3	2+3		
4	5		
5	5		
6	3		
7	2+3		
8	3+3+3+3		
Total	40		

1. (Bluman, 2001, 113) Thirty automobiles were tested for fuel efficiency (in miles per gallon). The following frequency distribution was obtained.

Class boundaries	f	F	F/n	
$(7.5,\ 12.5]$	3	3	0.10	
$(12.5,\ 17.5]$	5	8	0.267	
$(17.5,\ 22.5]$	16	24	0.80	
$(22.5,\ 27.5]$	4	28	0.933	
$(27.5,\ 32.5]$	2	30	1.00	

Fill in the blanks:

(a) 80% of the cars run **at most** ______________________miles per gallon.
(b) Most cars run **between** __________________________ miles per gallon.
(c) The **average** number of miles per gallon run by a car is ________________

2. Fill in the blanks:

(a) The positive skewness of a data set implies that the corresponding relative frequency curve has ______________________ tail.
(b) Eighty observations of the daily emission (in tons) of sulfur oxides from an industrial plant are obtained. The 20th and 21st ordered observations are 14.7 and 15.2 (Johnson, 2005, 34). The 25th percentile of the sample of 80 observations is ____________________.

3. (Johnson, 2005, 40)Consider a sample of 80 observations of the daily emission (in tons) of sulfur oxides from an industrial plant. The following summary statistics are : $\sum yf = 1512$ and $\sum yf^2 = 31008$.

a. What is the average amount of daily emission (in tons)?
b. Calculate coefficient of variation, and comment on the value.
[5 = 2+3 Marks]

4. Suppose that we are looking for a particular rare book and there are two places we can look: the bookstore and the antique store. Assume the probability that the bookstore has it is 0.20, the probability that the antique store has it is 0.30, and the probability that the bookstore and the antique store have it is 0.12. Find the probability that we will be able to locate the book at one place or the other. [5 Marks]

5. A shipment of 20 digital voice recorders contains 5 that are defective. If two of them are randomly inspected, what is the probability that both of them will be defective? [5 Marks]

6. A panel of meteorological and civil engineers studying emergency evacuation plans for a city in the event of hurricane has estimated that it would take between 13 and 18 hours to evacuate people living in low-lying land with the probabilities shown in the following table:

Time to evacuate nearest hour	probability

13	.04
14	.25
15	.40
16	.18
17	.10
18	.03

What is the probability a person will evacuate within 17 hours? [3 Marks]

7. The following table gives the probabilities that a certain computer will malfunction 0, 1, 2, 3, 4 times on a day:

x	0	1	2	3	4
$f(x)$	0.15	0.30	0.25	0.15	0.15

Find the mean and standard deviation of the number of times a computer malfunction. [5 = 2+3 Marks]

8. The radius of a flat metal disk manufactured by a factory is a random variable with density function

$$f(x) = \begin{cases} 2, & 4 < x < 4.5 \\ 0, & \text{otherwise} \end{cases}$$

a. What is the probability that radius of such a flat disk chosen at random is at least 2.1?
b. What is the expected radius of the disk?
c. What is the standard deviation of the radius of the disk?
d. What is the probability that the area of such a flat disk chosen at random is at least 4.41π ? (Note that the area of a circle is πx^2)

FORMULAE FOR STAT 319

A. Descriptive Statistics (for Samples)

***A.1 Mean** and **variance** are* $\bar{y} = \frac{1}{n} \sum y$ *and*

$$s^2 = \frac{TSS}{n-1} \text{ where } TSS = \sum (y-\bar{y})^2 = \sum y^2 - \frac{1}{n}\left(\sum y\right)^2 .$$

A.2 Quartiles: $R_\alpha = \alpha \frac{1+n}{100} = i + d, \quad \alpha = 1,\ 2,\ \cdots, 99;$

$$P_\alpha = (1-d)y_{(i)} + dy_{(i+1)}$$

A.3 Mean** and the **variance** for **grouped data:

$$\bar{y} = \frac{1}{n} \sum yf \ , \ s^2 = \frac{TSS}{n-1}, \quad TSS = \sum y^2 f \ - \ \frac{1}{n}\left(\sum yf\right)^2 .$$

A.4 Coefficient of Variation : $CV = s / \bar{y}$.

A.5 Coefficient of Skewness : $CS = \frac{\bar{y} - \tilde{y}}{s/3}$.

B. Glossary of Probability of Set Events (Two Sets)

	Verbal Description of Event	*Probability*
B.1	A *but not* B (*=Only* $A = A$ *alone*)	$P(A \cap \overline{B}) = P(A) - P(A \cap B)$
B.2	B *but not* A (*=Only* $B = B$ *alone*)	$P(\overline{A} \cap B) = P(B) - P(A \cap B)$
B.3	*None (=Neither* A *nor* B)	$P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$
B.4	*Exactly one*	$P(A \cap \overline{B}) + P(\overline{A} \cap B)$
B.5	*Both* (*=Exactly two = Two*)	$P(A \cap B) = 1 - P(\overline{A \cap B}) = 1 - P(\overline{A} \cup \overline{B})$ $P(A \cap B) = P(B)P(A \mid B) = P(A)P(B \mid A)$ $P(A \cap B) = P(A)P(B)$ iff A and B are independent
B.6	*Not both*	$P(\overline{A \cap B}) = 1 - P(A \cap B)$
B.7	A given B	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ for $P(B) \neq 0$ $P(A \mid B) = P(A)$ iff A and B are independent

B.8	*At least one of the two (= A or B)*	$P(A\cup B)=P(\text{exactly one})+P(\text{exactly two})$ $=P(A\cap\bar{B})+P(\bar{A}\cap B)+P(A\cap B)$ $=P(A)+P(B)-P(A\cap B)$ $=1-P(\overline{A\cup B})=1-P(\bar{A}\cap\bar{B})$

Also for Independence: $P(A\mid\bar{B})=P(A\mid B)=P(A);\ P(B\mid\bar{A})=P(B\mid A)=P(B)$

For intersection of three sets A, B and C *, the following results are important:*

B.9 $P(\text{none})+P(\text{at least one})=1$

B.10 $P(\text{none})=P(\bar{A}\cap\bar{B}\cap\bar{C})$
$=P(\bar{A})P(\bar{B})P(\bar{C})$ iff independent

B.11 $P(\text{at least one})=P(A\cup B\cup C)$
$=P(A)+P(B)+P(C)-[P(A\cap B)+P(A\cap C)+P(B\cap C)]+P(A\cap B\cap C)$

C. Discrete Probability Distributions

C.0a $\mu=E(Y)=\sum yp(y),\quad (p89)$

C.0b $E\left(Y^2\right)=\sum y^2\ p(y),\ \ \sigma^2=E(Y-\mu)^2=E\left(Y^2\right)-\mu^2,\ (p96)$

D. Continuous Probability Distributions

For a continuous random variable Y with pdf $f(y)$

$D.0\quad \int_{-\infty}^{\infty} f(y)dy=1;\ P(a<Y<b)=\int_a^b f(y)dy;\ P(Y\le u)=\int_{-\infty}^{u} f(y)dy$

$D.0a\quad \mu=E(Y)=\int_{-\infty}^{\infty} yf(y)dy,\ \ (p89)$

$D.0b\quad E\left(Y^2\right)=\int_{-\infty}^{\infty} y^2 f(y)dx,\ \ \sigma^2=E(Y-\mu)^2=E\left(Y^2\right)-\mu^2,\ \ (p96)$

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8	3+2+2+1		
Total	40		

1. (Johnson, 2005, 18) Consider a sample of 80 observations of the daily emission (in tons) of sulfur oxides from an industrial plant. The following frequency distribution was obtained.

Class boundaries	f	F	F/n	
[5, 9)	3	3	0.0375	
[9, 13)	10	13	0.1625	
[13, 17)	14	27	0.3375	
[17, 21)	25	52	0.65	
[21, 25)	17	69	0.8625	
[25, 29)	9	78	0.975	
[29, 33)	2	80	1.00	

Fill in the blanks:

a. 65% of the emission of sulfur oxides are less than _______tons per day.
b. Most of the time daily emission of sulfur oxides run between _________ tons per day.
c. The average number of miles per gallon run by a car is ________________

2. Fill in the blanks:

a. The z-scores of a set of observations have mean _____ and variance ______.
b. The Empirical Rule says for a sample producing bell shaped relative frequency curve, __________% of the observations lie __________ two times standard deviation of the mean.

3. The following miles per gallon (mpg) obtained in test runs performed on urban roads with an intermediate size car:

20.5	22.4	23.2	19.2	20.5
22.7	23.2	21.4	20.8	19.4
21.2	22.3	21.1	20.9	21.5
20.3	21.7	22.2	23.1	23.0

a. What is the average mileage?
b. Calculate coefficient of skewness, and comment on the value. [2+3 Marks]

4. The probability that an integrated circuit chip will have defective etching is 0.12, the probability that it will have a crack defect is 0.29, and the probability that it has **both** defects is 0.07.

(a) What is the probability that a newly manufactured chip will have either an etching **or** a crack defect?
(b) What is the probability that a newly manufactured chip will have neither defect? [$m+(5-m)=5$ Marks]

5. A certain person encounters three traffic lights when driving to work. Suppose that the following represent the probabilities of the total number or red lights that he has to stop for: P{ 0 red lights}=0.14, P{ 1 red lights}=0.36, P{ 2 red lights}=0.34
P{ 3 red lights}=0.16

a) What is the probability that he stops for at least one red light when driving work?
b) What is the probability that he stops for more that one red light?

[3+2 Marks]

6. A shipment of 25 digital voice recorders contains 5 that are defective. If two of them are randomly inspected, what is the probability that both of them will be defective? [5 Marks]

7. Consider the random variable "the number of tankers docked at a certain time at a certain seaport." Based on experience, suppose the probability distribution of the number of tankers is known to be as shown below:

Number of tankers docked	0	1	2	3	4
Probability	.17	.24	.30	.21	.08

(a) Find the probability that more than two tankers docked at a certain time.
(b) Find the average number of tankers at dock.
(c) Find the standard deviation for the number of tankers at dock.
[2+2+3 = 7 Marks]

8. The content of magnesium in an alloy is a random variable, given by the following pdf (probability density function)

$$f(y) = \frac{y}{18} \qquad 0 \le y \le 6 .$$

(a) What is the probability that the content of magnesium in alloy is at most 3?
(b) Find variability of the content of magnesium.
(c) Evaluate the following integral, and explain it.

$$\int_0^6 y\, f(y) dy$$

(d) Evaluate the value of κ from the following integral, and explain it.

$$\int_\kappa^6 y\, f(y) dy = 1/2$$

[3+3+3+3]